AN EMPIRICAL FRAMEWORK FOR MATCHING WITH IMPERFECT COMPETITION

MONS CHAN*, KORY KROFT†, ELENA MATTANA‡, ISMAEL MOURIFIɃ

Abstract. This paper develops and estimates an equilibrium model of wage-setting that incorporates rich two-sided heterogeneity and strategic interactions. We provide a tractable characterization of the model equilibrium and demonstrate its existence and uniqueness. This characterization of the equilibrium allows us to derive comparative statics and to gauge the relative contributions of worker skill, preference for amenities, and strategic interactions on equilibrium wages. Using instrumental variables and exploiting firm optimization, we establish identification of labor demand and supply parameters and estimate them using matched employer-employee data from Denmark. We use our estimated structural model to perform counterfactual analyses to provide a quantitative evaluation of the main sources of wage inequality in Denmark and assess the role of strategic interactions.

Keywords: Compensating differentials, Inequality, Imperfect competition, Matching equilibrium, Oligopsony, Strategic interactions, Sorting.

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* Department of Economics, Queen's University. mons.chan@queensu.ca.

† Department of Economics, University of Toronto, & NBER. kory.kroft@utoronto.ca.

‡ Department of Economics and Business Economics Aarhus University. emattana@econ.au.dk.

⊛ Department of Economics, Washington University in St. Louis & NBER. ismaelm@wustl.edu.

1. Introduction

It is widely recognized that workers with similar skills earn different wages depending on the firms they work for (Abowd, Kramarz and Margolis, 1999). Recent evidence has also shown that high-wage workers tend to work at high-wage firms. These empirical regularities are difficult to rationalize through the lens of the standard competitive model of wage determination, particularly given the observation that in many local labor markets the number of potential employers is relatively small, suggesting a role for concentration.[1] A recent literature has developed models to investigate the role of labor market power in generating firm-specific pay premia and worker sorting.

One strand of this literature has focused on models that allow for rich heterogeneity across workers and firms, but assumes that firms are "atomistic", ruling out strategic interactions in wage-setting (Card et al., 2018, Lamadon, Mogstad and Setzler, 2022, Azar, Berry and Marinescu, 2022). While these models can explain firm-level pay premia and worker sorting, they cannot explain the link between market structure, employer concentration and wages.

Another strand of the literature has considered models with strategic interactions in wage-setting (Berger, Herkenhoff and Mongey, 2022; Jarosch, Nimczik and Sorkin, 2024). However, these frameworks typically abstract from skill heterogeneity across workers and thus cannot capture earnings differences arising from standard competitive forces such as human capital or match-specific production complementarities—commonly referred to as Roy sorting. Thus, they cannot explain why high-wage workers sort into high-wage firms and the distributional consequences of monopsony and market power.[2]

When both two-sided heterogeneity and strategic interactions are present, neither modeling approach on its own can fully account for worker and firm outcomes. However, technical difficulties have limited progress in the literature. As Lamadon, Mogstad and Setzler (2022) write: if local markets are segmented by geography or location, then strategic interactions can play an important role, but "*identification of such interaction effects is challenging with two-sided heterogeneity*".

In this paper, we develop and estimate an equilibrium model of wage-setting that incorporates two-sided heterogeneity and strategic interactions, often referred to as "oligopsony". On the theory side, we provide a tractable characterization of the model equilibrium and demonstrate its existence and uniqueness. This characterization delivers comparative statics linking firm-specific technology shocks to wages and enables counterfactuals that parse the

[1] A growing number of studies have documented a link between concentration and wages. See Arnold (2021), Prager and Schmitt (2021), Benmelech, Bergman and Kim (2022), Rinz (2022), Azar, Marinescu and Steinbaum (2022), and Schubert, Stansbury and Taska (2024).

[2] He and le Maire (2022) study mergers and acquisitions in Denmark and show that *high-wage* managers were replaced in target establishments of mergers. This illustrates the importance of having a framework that accounts for worker heterogeneity within establishments. This is consistent with the view of mergers emphasized in Shleifer and Summers (1987).

contributions of worker skill, preferences for amenities, and strategic interactions on equilibrium outcomes. On the empirical side, we demonstrate how instrumental variables (IV) combined with firm-optimality conditions identify labor supply and labor demand parameters. We estimate the structural model using Danish firm balance sheet data linked to matched employee-employer data for 2001-2019. Our strategy leverages output market data to identify the marginal revenue product of labor (MRPL) and labor market data to estimate labor market elasticities—and thus the markdown of wages relative to the MRPL.

With the estimated model, we quantify the extent of labor market power and how it varies across workers and firms. We then run counterfactuals to isolate the role of strategic interactions in shaping equilibrium outcomes and to decompose the sources of wage dispersion. These analyses allow us to explore how different scenarios might affect wages, profits, concentration, and welfare, providing valuable insights into the mechanisms at play in the labor market. To our knowledge, this paper is among the first to develop a general equilibrium wage-setting model under imperfect competition, establish existence and uniqueness of the equilibrium, derive comparative statics, and explore various counterfactual scenarios.

Our equilibrium model of the labor market builds on existing models of monopsony in three key ways. First, we allow worker skills to be *multidimensional* and *match-specific* and we allow both deterministic and stochastic preferences over employer amenities.[3] This flexibility means that worker productivity can vary across firms, which allows for rich sorting patterns and captures the key features of the Roy model (Roy, 1951). Our setup generalizes existing approaches that restrict skills to be homogeneous across workers or allow only for one-dimensional heterogeneity. Our labor supply model also lets the wage coefficient vary with worker characteristics. This allows us to investigate how labor market power varies between workers and therefore contributes to our understanding of wage gaps, which was one of the original motivations for studying monopsony (Robinson, 1933). We illustrate this by empirically examining the gender wage gap and the role of market power.[4]

Second, we contribute methodologically by establishing general conditions on worker preferences and firm production technologies that guarantee existence (Theorem 1) and uniqueness (Theorem 2) of the equilibrium. Our uniqueness result is more general than the one obtained by Nocke and Schutz (2018) because it does not depend on the aggregative game structure that arises from the Nested Logit framework combined with a specific type of production function. Instead, our approach accommodates a broader class of preference shocks (and thus substitution patterns) and production functions, making it significantly

[3] We allow stochastic preferences to be correlated within local labor markets and the correlation parameter is market-specific. This implies that the markdowns in our model will vary across local markets. They will also vary across firms since they depend endogenously on firm-specific market shares. The heterogeneity in markdowns across workers and firms plays a key role in our counterfactuals as discussed below.

[4] Tino (2024) usefully applies our empirical framework to understand the pay gap between immigrants and natives and finds an important role for labor market power and worker sorting.

more flexible and widely applicable. We show that, under these conditions, globally convergent methods such as Gauss-Siedel or Jacobi iteration can be employed to find the unique equilibrium of the model (Proposition 1). This has an important advantage in empirical settings since it allows one to solve efficiently for the equilibrium, which is useful when performing counterfactual analyses. We use the same conditions to derive comparative statics and show analytically how the presence of strategic interactions amplifies the passthrough of productivity shocks (Proposition 2). Our results do not rely on any parametric assumptions on the preference shock distribution and, to our knowledge, are new to the literature.

Third, we demonstrate that our framework leads to a natural measure of market-level concentration known as the "generalized concentration index" (GCI), which is based on the generalized entropy concept introduced in Galichon and Salanié (2022). We show that under certain conditions, increases in market concentration can negatively impact social welfare. In the case of Nested Logit preferences, the GCI can be expressed as a weighted function that incorporates both "within-nest" concentration values, and a "between-nest" component. As noted by Maasoumi and Slottje (2003), this property of a concentration index is particularly valuable when accounting for heterogeneity across local markets. It allows for a more precise identification of the primary sources of concentration and facilitates the examination of the potentially varied effects of policy changes—such as minimum wage reforms—on specific markets, specific worker types and on overall market concentration. In contrast, the widely used Herfindahl-Hirschman Index (HHI) lacks this decomposability feature, limiting its effectiveness in such analyses. We leverage the GCI's flexibility to examine how strategic interactions shape concentration across worker groups and, in turn, social welfare.

Turning to the results of our empirical application, our estimates of labor supply indicate an average wage elasticity across worker types and establishments of 3.891. This implies that wages are marked down by roughly 22 percent below the MRPL. There is significant heterogeneity in labor supply elasticities—and therefore markdowns—across establishments and worker types: the 10th and 90th percentiles of the distribution of markdowns are equal to 0.692 and 0.849, respectively, implying that wages are marked down by 31 and 15 percent below the MRPL. Establishments that are larger in their local market tend to face smaller labor supply elasticities, and thus have more labor market power. In terms of heterogeneity across workers, our analysis shows that younger individuals (aged 35 or less) exhibit significantly higher elasticities compared to their older counterparts, implying that wages of younger men are marked down by less, on average, than those of older men. Younger women have larger elasticities than younger men, a pattern that reverses among older workers. Workers preferences for amenities vary significantly across establishments. We find that urban areas offer more valuable amenities than rural areas, while knowledge-based and manufacturing jobs have more valuable amenities than utilities, agricultural and food service jobs. We also find that high-value amenity establishments have more workers, pay lower wages, and have lower revenue on average. These results highlight how our framework is particularly well suited to studying the distributional consequences of monopsony and market power, as it allows for wage dispersion both within and across firms.

On the demand side, our findings indicate that higher educated and older workers are more productive than their less educated and younger peers, respectively. Our estimates indicate substantial variation in establishment-specific returns to scale and total factor productivity (TFP). The distribution of TFP is heavily skewed to the right with a 90-10 ratio of 3.128. We also find that the worker types in our empirical application are highly substitutable, and we quantify this using the Morishima (1967) elasticity of substitution.

Next, using our structural parameter estimates, we calculate the "establishment wage premium" – which we define as the component of the optimal wage that varies only by establishment – classify establishments on the basis of this premium, and examine the sorting patterns of workers across firms. We find clear evidence of sorting by skill: only 10 percent of workers in the bottom decile of the premium distribution are college educated, versus 38 percent in the top decile. In contrast, we do not find evidence of sorting based on gender.[5]

Finally, we evaluate market concentration using the GCI. Our findings reveal significant heterogeneity in concentration by worker type: local markets for highly educated workers are more concentrated than local markets for less educated workers, and local markets for women are more concentrated than local markets for men (at all education levels). Moreover, we show that one would obtain a very different ranking of concentration levels by worker type using the HHI instead of the GCI.

Our last step is to perform a series of counterfactual experiments to quantitatively examine the determinants of wage inequality, concentration, and welfare. To conduct each experiment, we begin with our estimated parameters and solve for the equilibrium distribution of wages and employment. To examine the role of the different model mechanisms, we restrict the model along some dimension and recalculate the new counterfactual equilibrium taking into account general equilibrium effects.

In order to assess how our findings compare to the two main strands of the labor literature outlined above, we consider two types of experiments. In the first, we remove worker skill heterogeneity and set the cross-type substitution parameters to their employment-weighted mean. This version of our model resembles the homogeneous worker oligopsony model of

[5]Gallen, Lesner and Vejlin (2019) examine sorting to workplace establishments by gender in Denmark in a sample that includes both the private and public sectors. They find that sorting accounts for about 15 percent of the gender wage gap between 2000-2010. On average, wages in the public sector are typically lower and women are more likely to work there. Since our sample excludes the public sector (because we use firm balance sheet data), this could explain why we do not find much evidence of sorting by gender.

Berger et al. (2022). Relative to the baseline economy, as expected, we find that the dispersion (across workers and firms) in the MRPL falls, which acts to reduce wage inequality. However, there is a second force in the model which dominates: the covariance across workers and firms between the MRPL and markdown, which is negative in the baseline economy, becomes negligible. This acts to increase overall wage inequality demonstrating the importance of allowing for heterogeneous markdowns across workers and firms.

In the second experiment, we shut down strategic interactions by forcing each firm to treat the market-level labor supply elasticity as exogenous. This "classic monopsony" setting resembles the one in Lamadon et al. (2022). In this scenario, the average wage (across workers and firms) increases by 2.2 percent. While in principle, removing strategic interactions reduce wage inequality through less dispersion in the markdown across firms, in practice it mainly leads to a shift in the mean wage. This is because the covariance between the MRPL and markdown across workers and firms is reduced, which counteracts the reduced dispersion in the markdown and increases overall wage inequality. A key advantage of our model is that we can examine heterogeneity across worker types. Our findings reveal that removing strategic interactions partially closes the wage gap between college-educated and non-college-educated workers. Part of this comes from a "bargaining effect" where wages are marked down by less for non-college-educated workers compared to college-educated workers. The remaining part comes from a "selection effect" where less-skilled workers are relatively more likely to sort to better firms. For gender, we find similar changes in markdowns and sorting between men and women. On the other hand, the welfare impacts reveal significant heterogeneity, with female and non-college educated workers gaining three and two times more than male and college-educated workers, respectively. We trace this gap to differential changes in market concentration, with the markets serving women experiencing larger declines.

We view the main contribution of this paper as the development of a new empirical model of imperfect competition that can explain the link between concentration and wages, the presence of firm pay premia, and the sorting of high-wage workers to high-wage firms. We also provide an algorithm for solving for the unique equilibrium. Our counterfactuals are a useful first step in demonstrating how our empirical framework can be applied practically to address important questions. Although outside the scope of our analysis, our framework is well suited for examining the efficiency *and* distributional consequences of labor market regulations such as minimum wages, tax and transfer policy, labor market institutions such as unions (as in Dodini, Salvanes and Willén, 2024) and mergers and acquisitions (as in Berger, Herkenhoff and Mongey, 2025). More broadly, it can be used to study counterfactual experiments that affect wages, amenities, and mobility patterns.

Related Literature. Our paper relates to and builds on the broader labor literature in the following ways. First, our paper adds to the growing literature on imperfect competition in labor markets.[6] Several studies have estimated firm-specific labor supply elasticities using the passthrough of firm-specific productivity or demand shocks under the assumption of monopsonistic competition and isoelastic labor supply curves, with estimates typically ranging between 4 and 6 (Kline et al., 2019, Dube et al., 2020, Huneeus, Kroft and Lim, 2021, Azar, Berry and Marinescu, 2022, Lamadon et al., 2022, Kroft et al., 2025). As our comparative statics show, this approach to recovering labor market power is not valid in the presence of strategic interactions.[7] In the general case, passthrough depends on the "super-elasticity" of labor supply to the firm.[8] Suggestive evidence on the importance of strategic interactions comes from Staiger, Spetz and Phibbs (2010) who use an exogenous change in wages in Veterans Affairs hospitals as a natural experiment and estimate a labor supply elasticity of 0.10. Their findings indicate that non-VA hospitals, who were not directly affected by the legislated change, responded by changing their own wages.

Several studies consider alternative approaches to recovering labor market power that are valid in the presence of strategic interactions. Berger, Herkenhoff and Mongey (2022) use an indirect inference approach that exploits changes in state-level corporate tax rates and find elasticities that range from ~ 5 (payroll-weighted average) to 9 (unweighted average). Yeh, Macaluso and Hershbein (2022) directly estimate plant-level markdowns in the manufacturing sector in the US using the so-called "production approach" and report a ratio of wages to MRPL of 0.65, implying that wages are marked down 35 percent below the MRPL. Our contribution to this literature is to provide an identification strategy for the structural labor supply elasticity that leverages the "quasi-labor supply function" (Berry, 1994) and instrumental variables. We also contribute to this literature by modeling heterogeneous elasticities across worker types, which allows us to empirically study the role of market power for pay gaps across observable groups of workers.

Our paper also adds to the literature that examines labor market sorting. Starting with Abowd, Kramarz and Margolis (1999) and subsequent research, there is extensive evidence

[6]Models of imperfect competition in the labor market have recently attracted interest because of their ability to explain various labor market features, such as wage dispersion for identical workers, the correlation between firm characteristics and wages, the lack of an impact of the minimum wage on employment, and the prevalence of gender and racial wage gaps. See Manning (2003) for an excellent overview of the literature.

[7]The model in Lamadon et al. (2022), which features monopsonistic competition with constant labor supply elasticities, predicts full passthrough of shocks in value-added per worker. This is rejected by the data: e.g., Kline et al. (2019) estimate a passthrough rate of 0.47 from log value added per worker to log wages. Models that allow for variable wage elasticity, such as the one considered in this paper, can rationalize this finding.

[8]See Kline (2025) for a rich discussion of passthrough and market power. It is also possible that passthrough depends on the super-elasticity in monopsonistic competition models that relax the assumption that the labor supply function is isoelastic.

that high-wage workers tend to work at high-wage firms. One source of this sorting is production complementarities. Recently, Borovičková and Shimer (2024) have shown that with only wage data, it is impossible to identify selection patterns unless auxiliary assumptions are imposed. One side of the labor literature sidesteps this identification challenge by assuming that workers are homogeneous and therefore do not allow for sorting (e.g., Berger, Herkenhoff and Mongey, 2022, Jarosch, Nimczik and Sorkin, 2024). Lamadon, Mogstad and Setzler (2022) allow for sorting due to production and amenities complementarities and establish identification by additionally leveraging firm data in the output market. We follow their approach and extend it in three ways: (i) we allow a more general functional form for production complementarities; (ii) we relax the assumption that worker types are perfect substitutes and estimate substitution parameters; and (iii) we relax the "full employment" assumption by introducing an outside option which corresponds to non-employment.[9]

Third, although we do not incorporate dynamic considerations, it relates to the search-and-matching literature that incorporates firm and worker heterogeneity. Search frictions are an important source of employer market power, as emphasized by Burdett and Mortensen (1998), Postel-Vinay and Robin (2002) and Taber and Vejlin (2020).[10] Our paper is most closely related to Taber and Vejlin (2020) in terms of the broader objective of decomposing wage inequality into a skill component, a preference component, and imperfect competition. While matching in most dynamic search models is one-to-one, our static framework features many-to-one matching.[11] Additionally, many search models in this literature cannot generate sorting in equilibrium, e.g., there is 0 covariance between worker and firm type in equilibrium, which is at odds with the allocation of workers to firms in many countries.

The outline of the rest of the paper is as follows: Section 2 presents our theoretical and empirical framework. Section 3 proposes a tractable characterization of the equilibrium wage and distributional matching function and discusses the existence and uniqueness of the equilibrium. Section 4 presents key comparative statics from the model. Section 5 discusses the identification and estimations of key parameters. Sections 6 and 7 present the results of our empirical application and our counterfactual analyses. The last section concludes. Proofs of the main results are collected in the Appendix.

2. THEORETICAL FRAMEWORK AND EMPIRICAL MODEL

Consider a static labor market with a large population of individuals divided into K finite types, $k \in \{1, .., K\} \equiv \mathcal{K}$. The type k can be thought of as being derived from multiple

[9] Lindner et al. (2022) use a two-factor firm-level CES production function with low- and high-skill work. They do not identify and estimate the elasticity of substitution but rather calibrate it using external estimates.
[10] Other papers in this literature include Lentz (2010); Lise, Meghir and Robin (2016); Hagedorn, Law and Manovskii (2017); Eeckhout and Kircher (2018); Lopes de Melo (2018), and Bagger and Lentz (2019).
[11] A paper that features a dynamic search model with many-to-one matching is Eeckhout and Kircher (2018).

underlying characteristics, discrete or continuous.[12] For each k, there is an infinite number of individuals of mass $\mathfrak{m}_k$ where $\sum_{k\in\mathcal{K}} \mathfrak{m}_k = 1$. We assume that there is a continuum of individuals of each type to simplify the analysis of the existence of a stable equilibrium and for tractability.[13] An individual i with characteristic k is denoted by k_i.

On the other side of the market, there is a finite set of firms, $\mathcal{J} \equiv \{1, .., J\}$. We do not require J to be large; pure monopsony is a special case. Firms differentiate workers at the k level, but within each type, individuals may differ in unobservable characteristics and firm preferences (unobserved to both firms and the econometrician). Each individual i chooses a firm (or non-employment), and each firm j sets wages for each worker type k.

Workers: Additive Random Utility Model (ARUM). Workers have heterogeneous preferences over firms. The potential utility of individual i of type k if offered a wage $w_{k_i j} \equiv w_{kj} \in [0, \infty)$ by firm j is given by:

$$U_{ij} = \beta_{kj} \ln w_{kj} + \ln u_{kj} + \epsilon_{ij},\ j \in \{1, ..., J\}, \tag{2.1}$$

where $u_{kj} \in (0, \infty)$ represents the deterministic non-pecuniary part of the worker's potential utility U_{ij}, and $\beta_{kj}^{-1} \in (0, \infty)$ can be interpreted as the standard deviation of ϵ_{ij} in log-dollars. The idiosyncratic payoff, ϵ_{ij}, is unknown to firms. Individual i's utility of being non-employed is given by:

$$U_{i0} = \beta_{k0} \ln w_{k0} + \epsilon_{i0}, \tag{2.2}$$

where $w_{k0} \in (0, \infty)$ is the non-employment benefit which we assume is an observable exogenous predetermined outcome.[14]

A type-k worker takes wages as given and has no market power over firms. Thus, given the potential wage streams $\{w_{kj}\}_{0\leq j\leq J}$, individual i chooses according to:

$$U_i \quad = \quad \max\{U_{i0}, U_{i1}, ..., U_{iJ}\} = \max_{j\in\mathcal{J}\cup\{0\}} \{v_{kj} + \epsilon_{ij}\},$$

where $v_{kj} \equiv \beta_{kj} \ln w_{kj} + \ln u_{kj}$ and $v_{k0} \equiv \beta_{k0} \ln w_{k0}$. Denoting $v_{k\cdot} \equiv (v_{k0}, v_{k1}, ..., v_{kJ})'$ and $v \equiv (v_{1\cdot}', ..., v_{K\cdot}')'$, expected utility is given by the *social surplus function* (McFadden, 1977; Manski and McFadden, 1981):

$$G_{k\cdot}(v_{k\cdot}) = \mathbb{E}\Big[\max_{j\in\mathcal{J}\cup\{0\}} \{v_{kj} + \epsilon_{ij}\}\Big]. \tag{2.3}$$

[12] In practice, each continuous characteristic (or discrete characteristic with unbounded support) $X_d : d \in \mathcal{D}$ is transformed into a discrete random variable $\mathbf{k}_d$ with realization k_d and with finite support $\mathcal{K}_d$. Each discrete variable with finite support X_d is just relabeled $\mathbf{k}_d$. The total number of types is therefore $K = K_1 \times ... \times K_{|\mathcal{D}|}$.

[13] With a finite population, there is almost always a profitable deviation which may complicate the analysis of the existence of a stable equilibrium.

[14] Note that we have implicitly used a location normalization when defining potential utility. Equivalently, we could have written utility as $U_{ij} = \ln \tilde{u}_{kj} + \beta_{kj} \ln w_{kj} + \epsilon_{ij}$, for $j \in \mathcal{J} \cup \{0\}$. However, since $\tilde{u}_{kj}$ and $\tilde{u}_{k0}$ cannot be separately identified, we adopt the convention that $u_{kj} = \frac{\tilde{u}_{kj}}{\tilde{u}_{k0}}$.

To define the optimal choice probabilities, we impose the following regularity assumption:

Assumption 1 (Independence and absolute continuity)**.** *The joint distribution function of ϵ (i) is independent of v for all $v \in \mathcal{V} \subseteq \mathbb{R}^{K(J+1)}$, and (ii) is absolutely continuous respect to the Lebesgue measure on $\mathbb{R}^{K(J+1)}$.*

Under Assumption 1, the Williams-Daly-Zachary theorem shows that:[15]

$$\mathbb{P}\big(v_{kj} + \epsilon_{ij} \geq v_{kj'} + \epsilon_{ij'} \text{ for all } j' \in \mathcal{J} \cup \{0\} \equiv \mathcal{J}_0\big) = \frac{\partial G_{k\cdot}(v_{k\cdot})}{\partial v_{kj}}, \tag{2.4}$$

and therefore, the labor supply function is given by:

$$(\ell_{kj})^s = m_k \frac{\partial G_{k\cdot}(v_{k\cdot})}{\partial v_{kj}}, \tag{2.5}$$

where $(\ell_{kj})^s$ represents the number of type-k workers that prefer firm j at the wage w_{kj}. Equation (2.5) provides a general form of labor supply that does not imposes a parametric distribution on idiosyncratic preferences and allows arbitrary correlation among them.

Firms: Wage-Posting framework. Each firm j has a production function given by $F^j(\ell_{\cdot j})$ where $\ell_{\cdot j} \equiv (\ell_{1j}, ..., \ell_{kj})$. For simplicity, we ignore capital and intermediate inputs and impose regularity conditions on the production function.

Assumption 2. *We assume the firms' production functions $F^j(.); j \in \mathcal{J}$ to be (a) twice continuously differentiable, (b) non-constant and non-decreasing in each of its arguments, to have bounded partial derivatives, and to have zero production with zero labor inputs, i.e., $0 \leq F_k^j(\ell_{\cdot j}) \equiv \frac{\partial F^j(\ell_{\cdot j})}{\partial \ell_{kj}} \leq \bar{F}' < \infty$ $\forall k \in \mathcal{K}$ and $F^j(0) = 0$.*

We adopt the Bertrand-Nash assumption where each firm j chooses its optimal wage for worker type k taking other firms' wages as given. Let $Q_j > 0$ denote the minimum acceptable output for each firm. Given the labor supply function (equation (2.5)) and Q_j, firm j's best response is obtained as follows:

$$\min_{w_{kj}} \sum_{k \in \mathcal{K}} w_{kj} \ell_{kj} \quad s.t. \;\; F^j(\ell_{\cdot j}) \geq Q_j, \;\; w_{kj} \geq 0$$

where

$$\ell_{kj} = m_k \frac{\partial G_{k\cdot}(v_{k\cdot})}{\partial v_{kj}}, \;\; (k,j) \in (\mathcal{K} \times \mathcal{J}). \tag{2.6}$$

Let $s_{kj} \equiv \frac{\ell_{kj}}{m_k}$ denote the share of type-k workers employed at firm j. Under Assumptions 1 and 2, at an interior solution the optimal wage is given by

$$w_{kj} = \lambda_j F_k^j(\ell_{\cdot j}) \frac{\mathcal{E}_{kj}}{1 + \mathcal{E}_{kj}}, \;\; \forall\, (k,j) \in \mathcal{K} \times \mathcal{J}, \tag{2.7}$$

[15]See alternatively Lemma 2.1 in Shi, Shum and Song (2018).

where $\mathcal{E}_{kj} \equiv \frac{w_{kj}}{\ell_{kj}} \frac{\partial \ell_{kj}}{\partial w_{kj}} \equiv \frac{w_{kj}}{s_{kj}} \frac{\partial s_{kj}}{\partial w_{kj}}$ is the elasticity of labor supply at the optimal wage.[16] More generally, we define the cross-wage elasticity of labor supply as $\mathcal{E}_{kjl} \equiv \frac{w_{kl}}{s_{kj}} \frac{\partial s_{kj}}{\partial w_{kl}}$ and use the shorthand notation $\mathcal{E}_{kjj} \equiv \mathcal{E}_{kj}$. Under Assumption 1, the social surplus function is convex implying that $\mathcal{E}_{kj} \geq 0$. The term λ_j is the lagrange multiplier representing the marginal cost of production that a profit-maximizing firm equates to its marginal revenue product. We assume that λ_j is bounded, $0 < \lambda_j < \bar{\lambda} < \infty$. Note that Assumption 2(b) ensures that $F_k^j(\ell_{\cdot j}) \geq 0$, so the optimal wage is always non-negative. The wedge between the wage and the MRPL, $\frac{\mathcal{E}_{kj}}{1+\mathcal{E}_{kj}}$, is the markdown, i.e., the fraction of the MRPL paid to the worker. Let $\mathcal{C}^j \subseteq \mathcal{K}$ denote the set of worker types to whom firm j offers a strictly positive wage $w_{kj} > 0$, which according to our ARUM specification and Assumption 1, is equivalent to $s_{kj} > 0$. Assumption 2(b) and the optimality conditions ensure that $\mathcal{C}^j \neq \{\emptyset\}$ for all firms j in the market, where $\mathcal{C}^j \equiv \{k \in \mathcal{K} : s_{kj} > 0\} = \{k \in \mathcal{K} : w_{kj} > 0\}$.

Using equation (2.6), we can write the labor supply elasticity in terms of the social surplus function as follows:

$$\mathcal{E}_{kj} = \beta_{kj} \frac{\frac{\partial^2 G_{k\cdot}(v_{k\cdot})}{\partial^2 v_{kj}}}{\frac{\partial G_{k\cdot}(v_{k\cdot})}{\partial v_{kj}}}.$$

Therefore, each firm plays its best response strategy taking other firms' wage as given. Their posted wage stream is given as follows:

$$w_{kj} = \lambda_j \beta_{kj} F_k^j(\ell_{\cdot j}) \frac{\frac{\partial^2 G_{k\cdot}(v_{k\cdot})}{\partial^2 v_{kj}}}{\frac{\partial G_{k\cdot}(v_{k\cdot})}{\partial v_{kj}} + \beta_{kj} \frac{\partial^2 G_{k\cdot}(v_{k\cdot})}{\partial^2 v_{kj}}} \forall\ (k, j) \in \mathcal{C}^j \times \mathcal{J}. \tag{2.8}$$

So far, we have described the behavior of each side of the market. Now, we define an equilibrium for this many-to-one employee-employer matching model. Let $\mathbb{R}_{\geq 0}$ denote $\{x \in \mathbb{R} : x \geq 0\}$ and $\mathbb{R}_{>0} \equiv \{x \in \mathbb{R} : x > 0\}$.

Definition 1. *Consider workers that have preferences which are of the ARUM form, i.e., equation (2.1), and firms that have production functions which satisfy Assumption 2. An equilibrium outcome (s, w) consists of a distributional worker-firm matching function and an equilibrium wage equation such that $w \equiv (w_{10}, ..., w_{KJ}) \in (\mathbb{R}_{\geq 0})^{K(J+1)}$ and $s \equiv (s_{10}, ..., s_{KJ}) \in [0, 1]^{K(J+1)}$ are optimal for workers and firms (workers maximize their utilities, firms set their optimal wages in a Bertrand oligopsony model), and the following population constraint holds*

$$\sum_{j \in \mathcal{J}} s_{kj} + s_{k0} = 1, \quad k \in \mathcal{K}. \tag{2.9}$$

Under Assumptions 1 and 2, the equilibrium outcome is equivalent to satisfying equations (2.6), (2.8) and (2.9).

[16]By convention and to ease the notation, we consider that $\mathcal{E}_{kj} = 0$ when $s_{kj} = 0$. The details of the derivation of equation (2.7) are in Appendix A.1.

2.1. **Social Welfare, Generalized Entropy and Market Concentration.** In this section, we define social welfare and establish a link with market concentration. We assume that total firm profits in the economy are redistributed to a group $\mathcal{R} \subseteq \mathcal{K} \times \mathcal{J}_0$ of workers, in proportion to their equilibrium wages (non-employment benefit for the non-employed). Formally, we have:

$$\sum_{j=1}^{J} \underbrace{\left(\lambda_j F^j \left(\ell_{\cdot j} \right) - \sum_{k=1}^{K} w_{kj} \ell_{kj} \right)}_{\pi_j} = \sum_{(k,j) \in \mathcal{R}} \phi(s, w; \lambda, \mathcal{R}) w_{kj} \ell_{kj}, \tag{2.10}$$

where $\lambda = (\lambda_1, ..., \lambda_J)'$.[17] Collect all primitives parameters of the model into a vector Ξ. The social welfare function for the many-to-one matching model is defined by the following utilitarian function:[18]

$$\mathcal{W}(\Xi, \lambda, \mathcal{R}) = \sum_{k=1}^{K} m_k G_{k\cdot}(\widetilde{v}_{k\cdot}) \tag{2.11}$$

where

$$\widetilde{v}_{kj} \equiv \begin{cases} \beta_{kj} \ln \left\{ w_{kj}(1 + \phi(s, w; \lambda, \mathcal{R})) \right\} + \ln u_{kj} = v_{kj} + \beta_{kj} \ln(1 + \phi(s, w; \lambda, \mathcal{R})), \text{ if } (k,j) \in \mathcal{R} \\ v_{kj}, \text{ if } (k,j) \notin \mathcal{R}. \end{cases}$$

With this representation, all agents that are not included in $\mathcal{R}$ are excluded from the profit sharing. Let $G^*_{k\cdot}(s_{k\cdot})$ denote the *convex conjugate* or *Legendre-Fenchel transform* of $G_{k\cdot}(v_{k\cdot})$. Convex duality implies the following relationship between the adjusted social surplus function and its convex conjugate:[19]

$$G_{k\cdot}(\widetilde{v}_{k\cdot}) = \sum_{j=0}^{J} \widetilde{v}_{kj} s_{kj} - G^*_{k\cdot}(s_{k\cdot}). \tag{2.12}$$

Using the above relationship (2.12), the welfare function becomes:

$$\mathcal{W}(\Xi, \lambda, \mathcal{R}) = \sum_{(k,j) \in \mathcal{K} \times \mathcal{J}_0} m_k v_{kj} s_{kj} + \ln[1 + \phi(s, w; \lambda, \mathcal{R})] \sum_{(k,j) \in R} m_k \beta_{kj} s_{kj} - \sum_{k=1}^{K} m_k G^*_{k\cdot}(s_{k\cdot}) \tag{2.13}$$

where $\phi(s, w; \lambda, \mathcal{R}) = \frac{\sum_{j=1}^{J} \pi_j}{\sum_{(k,j) \in \mathcal{R}} w_{kj} \ell_{kj}}$.

[17]This means that profit-sharing is such that agents receiving the transfer will have the following ex-post utility: $\widetilde{U}_{ij} = \ln u_{kj} + \beta_{kj} \ln \left\{ w_{kj}(1 + \phi(s, w; \lambda)) \right\} + \epsilon_{ij}$.

[18]This welfare function extends and generalizes the one considered in Lamadon, Mogstad and Setzler (2022) that assumes $\beta_{kj} = \beta, \mathcal{R} = (\mathcal{K} \times \mathcal{J})$, and full employment, i.e., $s_{k0} = 0$ for all $k \in \mathcal{K}$.

[19]See Galichon and Salanié (2022) for more detailed discussion.

The welfare function in equation (2.13) is the sum of two components: (i) the deterministic gains accruing in equilibrium to all agents— via wages, amenity preferences, and redistributed firm profits— and (ii) a term capturing market randomness driven by unobserved heterogeneity in workers' utilities. When ϵ follows the Logit distribution, $-G^*_{k\cdot}(s_{k\cdot})$ coincides with Shannon entropy, a natural measure of statistical disorder in information theory, which here reflects the degree of *incomplete information* in the market.[20] This structure lets us define a market concentration index directly tied to welfare: the *generalized concentration index (GCI)* as $GCI(s_{k\cdot}) \equiv e^{G^*_{k\cdot}(s_{k\cdot})}$. Social welfare is then given by:

$$\mathcal{W}(\Xi, \lambda, \mathcal{R}) = \sum_{(k,j)\in\mathcal{K}\times\mathcal{J}_0} m_k v_{kj} s_{kj} + \ln[1+\phi(s,w;\lambda,\mathcal{R})] \sum_{(k,j)\in R} m_k \beta_{kj} s_{kj} - \sum_{k=1}^{K} m_k \ln GCI(s_{k\cdot}) \quad (2.14)$$

This latter equation allows one to assess how changes in local concentration affect social welfare. It demonstrates that social welfare is a decreasing function of the GCI, holding fixed the deterministic gains from matching.

3. EXISTENCE AND UNIQUENESS OF EQUILIBRIUM

In this section, we establish existence and uniqueness of equilibrium in our many-to-one matching model. Moreover, we establish conditions under which there exist globally convergent methods to solve for the unique equilibrium.

Theorem 1. *[Existence] Under Assumptions 1 and 2, an equilibrium exists.*

The proof presented in Appendix B.1 mainly relies on Brouwer's fixed-point theorem.

In a many-to-one matching model with a finite number of firms and unrestricted strategic interactions, a shock to one firm's productivity in equilibrium could affect employment and wages of other firms in the economy. Hence, there could exist multiple equilibria in this environment.[21] We now characterize a set of shape restrictions on the firms' production functions and the labor supply elasticities that ensures the existence of a unique equilibrium. We define the k-type "cross-wage super-elasticities" of labor supply as $\zeta_{kjl} \equiv \frac{w_{kl}}{\mathcal{E}_{kj}}\frac{\partial \mathcal{E}_{kj}}{\partial w_{kl}}$. ζ_{kjl} is the elasticity of the labor supply elasticity of type-k worker at firm j with respect to the type k wage at firm l, w_{kl}. In absence of strategic interactions, $\zeta_{kjl} = 0$ for $j \neq l$. The term $\zeta_{kjj} \equiv \zeta_{kj}$ is the so-called "super-elasticity" discussed in Klenow and Willis (2016), Nakamura and Zerom (2010), and Edmond, Midrigan and Xu (2023).

[20]In their one-to-one matching model with perfect competition, Caldwell and Danieli (2024) make use of the continuous version of the Shannon entropy index as a measure of industrial concentration.

[21]Card et al. (2018) also discuss the complications that arise in the presence of multiple equilibria in a framework with a finite number of firms.

Assumption 3 (Shape Restrictions)**.** *(i) [cross-wage super-elasticity] Assume that the social surplus function is such that whenever all others entries $w_{kl'}$ (for $l' \neq l$) remain constant we have for all $k \in \mathcal{K}$*

$$\zeta_{kjl} \begin{cases} \leq 0, & \text{if } l = j \\ \geq 0, & \text{if } l \in \mathcal{J}_0 \setminus \{j\} \end{cases}$$

(ii) [Production function] The production function takes the following functional form:

$$F^j(\ell_{\cdot j}) = \sum_{k \in \mathcal{K}} h_k(\ell_{kj}),$$

where h is a $C^2(\mathbb{R})$ function such that $h_k'(x) \geq 0$ and $h_k''(x) \leq 0$.

Assumption 3(i) imposes a sign restriction on the cross-wage super-elasticities. The sign restriction requires that when firm j increases the wage of a type k worker, the labor supply elasticity decreases; conversely, it increases when another firm l increases the type k wage. This sign restriction is satisfied for a wide class of error distributions including the widely-used Nested Logit. Assumption 3(ii) allows the production function to have decreasing or constant returns to scale, and a non-constant marginal rate of substitution.

Theorem 2. *[Existence and Uniqueness] Under Assumptions 1, 2, and 3, an equilibrium exists and it is unique.*

The proof presented in Appendix B.2 relies on the observation that the mapping induced by equation (2.7) is globally invertible, since its Jacobian matrix is positive diagonally dominant.

Assumption 3 is only a sufficient condition for the uniqueness of the equilibrium. In the proof, we also discuss the case where Assumption 3(ii) does not hold, i.e., the production function is not additive separable. We show that the equilibrium can also be unique under an additional sign restriction on a component involving the production function partial mixed-derivatives and the cross-wage labor supply elasticity, i.e., $F^j_{kl}(\ell_{\cdot j}) \equiv \frac{\partial^2 F^j(\ell_{\cdot j})}{\partial \ell_{kj} \partial \ell_{kl}}$ and $\mathcal{E}_{kjl}$. This restriction could be tested if the primitive parameters of this model are known.

Our uniqueness result is valid for a broad class of preference shocks. Prior frameworks—Card et al. (2018), Lamadon, Mogstad and Setzler (2022), and Berger, Herkenhoff and Mongey (2022)— typically assume Nested Logit shocks. In Online Appendix C.1, we analyze the Nested Logit case and show that the shape restriction in Assumption 3(i) holds. Hence, uniqueness also obtains under Nested Logit provided the production function satisfies Assumption 3(ii). For comparison, Nocke and Schutz (2018) establish uniqueness in an oligopoly model with Nested Logit demand via a fixed-point argument; their result hinges on the Nested Logit structure with a linear production function which implies that their model can be framed as an aggregative game. This reformulation expresses the equilibrium as a fixed-point problem involving only market-level and nest-level indices (aggregators).

Our result is more general than that of Nocke and Schutz (2018): it does not require an aggregative game representation—a structure that in their setting arises from Nested Logit demand paired with a very specific production function. Instead, we allow a broader class of preference shocks (and thus substitution patterns) and production functions, yielding a substantially more flexible and widely applicable framework.

3.1. **Finding the Equilibrium: An Iterative Method.** Our uniqueness result implies that there exist globally convergent methods for recovering the unique equilibrium outcome (s, w). This is very important to solve for the model equilibrium to perform counterfactual analyses. In particular, we show that under Assumptions 1, 2, and 3 the nonlinear Gauss-Seidel or Jacobi iteration described below converges to the unique equilibrium. A key advantage of this result is that the Gauss-Seidel and Jacobi algorithms are easy to implement and can converge fairly rapidly, even with a very large system of equations.

Define the following object:

$$\delta_{kj}(w) \equiv w_{kj} - \lambda_j F_k^j\left(\ell_{\cdot j}(w)\right) \frac{\mathcal{E}_{kj}(w)}{1+\mathcal{E}_{kj}(w)}, \quad \forall (k,j) \in \mathcal{K} \times \mathcal{J}. \tag{3.1}$$

$\delta(w) = (\delta_{11}(w), ..., \delta_{KJ}(w)) : \mathbb{T}_\epsilon \subseteq \mathbb{R}^{KJ} \longrightarrow \mathbb{R}^{KJ}$, where $\mathbb{T}_\epsilon$ is a closed and bounded rectangular region.[22]

Algorithm 1 (Underrelaxed Gauss-Seidel Iteration)**.** *For $\xi \in (0,1]$:*

(1) Solve $\delta_{kj}(w_{11}^{t+1}, ..., w_{1J}^{t+1}, ..., w_{k,j-1}^{t+1}, w_{kj}, w_{k,j+1}^{t}, ..., w_{KJ}^{t}) = 0$ for w_{kj} holding all other components fixed.

(2) Set $w_{kj}^{t+1} = (1-\xi)w_{kj}^t + \xi w_{kj}$ and this for $kj = 11, ..., KJ$ and $t = 0, 1, ...$

Algorithm 2 (Underrelaxed Jacobi Iteration)**.** *For $\xi \in (0,1]$:*

(1) Solve $\delta_{kj}(w_{11}^{t}, ..., w_{1J}^{t}, ..., w_{k,j-1}^{t}, w_{kj}, w_{k,j+1}^{t}, ..., w_{KJ}^{t}) = 0$ for w_{kj} holding all other components fixed.

(2) Set $w_{kj}^{t+1} = (1-\xi)w_{kj}^t + \xi w_{kj}$ and this for $kj = 11, ..., KJ$ and $t = 0, 1, ...$

Proposition 1 (Convergence of the nonlinear Gauss-Seidel and Jacobi iteration)**.** *Suppose Assumptions 1, 2, and 3 hold. For $\xi \in (0,1]$ and any initial value $w^0 \in \mathbb{T}_\epsilon$ the nonlinear Gauss-Seidel or Jacobi iteration described in Algorithms 1 and 2 converges to the unique equilibrium wage w^{eq}. The equilibrium outcome is given by (w^{eq}, s^{eq}) with $s_{kj}(w^{eq}) = \frac{\partial G_{k\cdot}(v_{k\cdot})}{\partial v_{kj}}|_{v_{kj}=v_{kj}^{eq}}$ where $v_{kj}^{eq} \equiv \beta_{kj} \ln w_{kj}^{eq} + \ln u_{kj}$.*

The proof is in Appendix B.3.

[22]Please refer to the proof of Theorem 1 in Appendix B.1 for the complete definition of $\mathbb{T}_\epsilon$.

4. Comparative Statics

In the previous section, we introduced an efficient computational approach for conducting counterfactuals within our general framework. Analytical results for exogenous parameter changes are also valuable, as they illuminate the model's economic structure. In nonlinear systems, however, comparative statics are typically hard to obtain: applying the Implicit Function Theorem requires a closed-form inverse of the Jacobian associated with the mapping in equation (3.1). With strategic interactions, this is especially challenging—nonzero cross-wage elasticities render a closed-form inverse intractable as the number of firms grows. Here, we exploit special features of the Jacobian to derive informative bounds: we obtain closed-form comparative statics in duopsony and lower bounds in general oligopsony for the effect of a change in total factor productivity (TFP) on equilibrium wages. We provide comparative statics for the effect of changes in amenities and non-employment benefits on equilibrium wages in Online Appendix D.1.

Recall that the optimal wages is expressed as

$$w_{kj} = \lambda_j \underbrace{F_k^j(\ell_{\cdot j})}_{\mathrm{mpl}_{kj}} \underbrace{\frac{\mathcal{E}_{kj}}{1+\mathcal{E}_{kj}}}_{\mathrm{md}_{kj}}, \quad \forall\, (k,j) \in \mathcal{K} \times \mathcal{J}.$$

where mpl_{kj} and md_{kj} denote, respectively, the marginal productivity of labor and the markdown of firm j for a type-k worker. The elasticities of mpl_{kj} and md_{kj} with respect to the type k wage at firm l are given by:

$$\frac{\partial \ln \mathrm{mpl}_{kj}}{\partial \ln w_{kl}} = \underbrace{\frac{w_{kl}}{\ell_{kj}} \frac{\partial \ell_{kj}(w_{k\cdot})}{\partial w_{kl}}}_{\mathcal{E}_{kjl}} \underbrace{\left(\frac{F_{kk}^j}{F_k^j}\ell_{kj}\right)}_{1/\eta_{kj}}, \quad \frac{\partial \ln \mathrm{md}_{kj}}{\partial \ln w_{kl}} = \underbrace{\frac{1}{(1+\mathcal{E}_{kj}(w_{k\cdot}))}}_{1-\mathrm{md}_{kj}} \underbrace{\frac{w_{kl}}{\mathcal{E}_{kj}(w_{k\cdot})} \frac{\partial \mathcal{E}_{kj}(w_{k\cdot})}{\partial w_{kl}}}_{\zeta_{kjl}}.$$

In equilibrium, the marginal cost of output, λ_j, is equal to the marginal revenue product. For simplicity, we assume that all firms j are price takers on the output market, i.e., $\lambda_j = P_j$ where P_j is the market price which is exogenous.[23] Under this assumption, we can define the labor demand elasticity as the elasticity of the inverse marginal revenue product of labor curve, $\eta_{kj} \equiv F_k^j / \ell_{kj} F_{kk}^j$.[24]

The cross-wage elasticities $\mathcal{E}_{kjl}$, the cross-wage super-elasticities ζ_{kjl}, the markdowns md_{kj}, and the labor demand elasticities η_{kj} are the key statistics that collectively determine the effect of changes of model parameters on equilibrium wages. They are the key channels by which an exogenous shock at firm l affects firm j's equilibrium wage. Recall that under the atomistic firms assumption imposed in Card et al. (2018) and Lamadon, Mogstad and

[23] It is worth noting this restriction is not critical for deriving $\frac{\theta_l}{w_{kj}} \frac{\partial w_{kj}}{\partial \theta_l}$.

[24] Weyl and Fabinger (2013) define a similar object when analyzing the output market, although the relevant object in their setting is the output "supply elasticity".

Setzler (2022), $\mathcal{E}_{kjl} = 0$ for all $l \neq j$ and $\zeta_{kjl} = 0$ for all $l, j \in \mathcal{J}$. The equilibrium restriction entertained in Berger, Herkenhoff and Mongey (2022) relaxes the latter restrictions but still imposes that $\mathcal{E}_{kjl} = \zeta_{kjl} = 0$ for all firms l and j belonging to different local markets or groups. We do not impose such restrictions and thus provide a more general set of comparative statics.

Before presenting our main results, let us introduce the following shorthand notation for the derivative of the log wage of type-k workers at firm j with respect to log wages of type-k workers at firm l:

$$\psi_{k,jl} = \frac{\partial \ln \mathrm{mpl}_{kj}}{\partial \ln w_{kl}} + \frac{\partial \ln \mathrm{md}_{kj}}{\partial \ln w_{kl}} \equiv \frac{\mathcal{E}_{kjl}}{\eta_{kj}} + (1 - \mathrm{md}_{kj})\zeta_{kjl}.$$

In the next proposition, we examine how a positive TFP shock at firm l affects equilibrium wages. We assume that firm l's production function satisfies $F^l(.) = \check{\theta}_l \check{F}^l(.)$ where $\frac{\partial \check{F}^l(.)}{\partial \check{\theta}_l} = 0$, and $\check{F}^l(.)$ respects Assumption 3 (iii).

Proposition 2 (Comparative Statics)**.** *Consider that Assumptions 1, 2, and 3 hold. Let (s, w) denote the unique equilibrium outcome of our model. In a neighborhood of the equilibrium (s, w), the following (general equilibrium) comparative statics hold:*

(i) **Duopsony***: $\mathcal{J} = \{j, l\}$. If the firms' production functions have a multiplicative structure of the form $F^l(.) = \check{\theta}_l \check{F}^l(.)$ where $\frac{\partial \check{F}^l(.)}{\partial \check{\theta}_l} = 0$, then for any $k \in \mathcal{C}^j \cap \mathcal{C}^l$:*

$$\frac{\check{\theta}_l}{w_{kj}} \frac{\partial w_{kj}}{\partial \check{\theta}_l} = \frac{\psi_{k,jl}}{(1 - \psi_{k,jj})(1 - \psi_{k,ll}) - \psi_{k,jl}\psi_{k,lj}} \geq 0,$$
$$\frac{\check{\theta}_l}{w_{kl}} \frac{\partial w_{kl}}{\partial \check{\theta}_l} = \frac{(1 - \psi_{k,jj})}{(1 - \psi_{k,jj})(1 - \psi_{k,ll}) - \psi_{k,jl}\psi_{k,lj}} > 0.$$

(ii) **Oligopsony***: $J \geq 2$. If the firms' production functions have a multiplicative structure of the form $F^l(.) = \check{\theta}_l \check{F}^l(.)$ where $\frac{\partial \check{F}^l(.)}{\partial \check{\theta}_l} = 0$, then for any $k \in \mathcal{C}^j \cap \mathcal{C}^l$, we have:*

$$\frac{\check{\theta}_l}{w_{kj}} \frac{\partial w_{kj}}{\partial \check{\theta}_l} \begin{cases} \geq \frac{\mathcal{E}_{kjl}/\eta_{kj} + (1 - md_{kj})\zeta_{kjl}}{\left(1 - \mathcal{E}_{kj}/\eta_{kj} - (1 - md_{kj})\zeta_{kj}\right)(1 - \mathcal{E}_{kl}/\eta_{kl} - (1 - md_{kl})\zeta_{kl})} \geq 0 \ \ if\ j \neq l, \\ \geq \frac{1}{(1 - \mathcal{E}_{kl}/\eta_{kl} - (1 - md_{kl})\zeta_{kl})} > 0, \ \ if\ j = l. \end{cases}$$

where $\psi_{k,jl} \geq 0$ for $l \neq j$, and $\psi_{k,ll} \leq 0$.

The equations in part (i) show two key channels by which a productivity shock in firm l affects equilibrium wages in a duopsony market. The increase in firm l's TFP $\check{\theta}_l$ has a direct effect on mpl_{kl} and firm l raises w_{kl} through $\psi_{k,ll}$. In turn, this affects mpl_{kj} and md_{kj} through $\psi_{k,jl}$, and firm j responds to this change through $\psi_{k,jj}$ by raising w_{kj}. This succession of responses converges to higher equilibrium w_{kj} and w_{kl}.

In the general case ($J \geq 2$), part (ii) shows that strategic interactions amplify the impact of a firm-specific shock on equilibrium wages. The lower bound for passthrough is attained when strategic interactions are shut down. This encompasses two useful special cases. The first case is a setting with many local markets, each with a single dominant firm that internalizes wage effects locally but not on the aggregate wage index. Here, each firm j's labor-supply elasticity is variable and depends on its market share (a special case of Berger, Herkenhoff and Mongey, 2022). The second case is the monopsonistic competition framework of Lamadon, Mogstad and Setzler (2022), where all firms face constant labor-supply elasticities and the passthrough of TFP shocks to wages is likewise constant.[25]

To show how our comparative statics generalize prior special cases, Online Appendix C analyzes a Nested Logit Economy. We derive a formula for the passthrough of a TFP shock on wages and contrast it with the formulas in Card et al. (2018), Lamadon, Mogstad and Setzler (2022), and Berger, Herkenhoff and Mongey (2022), which impose restrictions on strategic interactions. In Section 6, we report estimates of the lower bound for TFP passthrough in the Nested Logit case (Online Appendix C.2, equation (C.4)) for Danish firms.

5. Econometric model: Identification and Estimation

In this section, we analyze identification of the structural parameters when preference shocks follow a parametric distribution. To this end, we focus on a specific case: the Nested Logit Economy, as considered in Card et al. (2018), Lamadon, Mogstad and Setzler (2022), Berger, Herkenhoff and Mongey (2022), and others. We begin by discussing the key implications of the Nested Logit Economy. Additional details are in Online Appendix C.

To allow unobserved worker preferences ϵ_{ij} to be correlated for certain classes of firms, we partition the J firms into G nests, where each nest is a local labor market. The g^{th} nest contains N_g firms. We allow the preference shocks, ϵ_{ij}, to be arbitrarily correlated within nests, i.e., $1/\sigma_{kg} = \sqrt{1 - corr(\epsilon_{ij}, \epsilon_{il})}$ for $j \neq l$ where for $(j, l) \in N_g$, and with $\sigma_{kg} \in [1, \infty)$. Each firm competes with every firm in the economy regardless of whether the firms belong to the same nest or not.

The labor supply elasticities and the cross-wage super-elasticities in the Nested Logit economy take the following form:

$$\mathcal{E}_{kj} = \beta_{kj}[\sigma_{kg} + (1 - \sigma_{kg})s_{kj|g} - s_{kj}] \quad \text{for } j \in N_g \tag{5.1}$$

$$\zeta_{kjl} = \beta_{kj}\left[(1 - \sigma_{kg})s_{kj|g}\frac{\mathcal{E}_{kjl|g}}{\mathcal{E}_{kj}} - s_{kj}\frac{\mathcal{E}_{kjl}}{\mathcal{E}_{kj}}\right] \tag{5.2}$$

with $s_{kg} = \sum_{j \in N_g} s_{kj}$, and $s_{kj|g} = \frac{s_{kj}}{s_{kg}}$ where $s_{kj|g}$ denotes the share of type k individuals employed by firm j as a fraction of the total nest share.

[25] All the lower bounds in Proposition 2 are sharp, meaning that there exists a data generating process under which these inequalities hold exactly with equality. In particular, the inequalities in Proposition 2(ii) hold as equalities when strategic interactions are assumed away. A formal proof of this statement is derived in Online Appendix D.1.

The atomistic firm assumption considered in Card et al. (2018) and Lamadon, Mogstad and Setzler (2022) implies that $(1-\sigma_{kg})s_{kj|g} - s_{kj} = 0$ for all $(k,j) \in \mathcal{K} \times \mathcal{J}$, and $g \in \{1,...,G\}$. With $\sigma_{kg} > 1$, this implies that $s_{kj|g} = s_{kj} = 0$. In other words, the atomistic firm assumption does not allow some firms to be dominant in their local market. If we observe some firms with a significant share of type k workers in their local market, i.e., $s_{kj|g} > \underline{s}$ for $\underline{s} > 0$, we can reject the atomistic firm assumption. Moreover, with $\sigma_{kg} > 1$, we always have that $[(1-\sigma_{kg})s_{kj|g} - s_{kj}] \leq 0$, which implies that the atomistic firm assumption leads to an overestimation of firms' labor supply elasticities and thus markdowns, as well as an overestimation of the super-elasticities. Thus, the atomistic firm assumption limits the effect of market power and imposes restrictions on the nature of strategic interactions. Conversely, Berger, Herkenhoff and Mongey (2022) impose that $s_{kj} = 0$ for all (k,j), but allow $(1-\sigma_{kg})s_{kj|g} \neq 0$ for some (k,j). In other words, firms can be dominant in their local market but not in the national market. This also leads to overestimation of the true markdowns and super-elasticities but with a lower bias than the one obtained under the atomistic firm assumption.

Thus far, we have considered a static model. We now assume that the econometrician has panel data linking workers to firms over time. We denote t the unit of time and let $t \in \{1,...,T\}$. For tractability, we assume that both the econometrician and firms observe worker type k.[26] Under this assumption, our identification approach can be summarized in two steps. First, we identify the labor supply parameters using instrumental variables (IV). Second, we identify the production function parameters by exploiting firm optimization together with an instrumental variables strategy. It is worth noting that our identification approach does not require solving the model equilibrium, so identification is robust to the existence of multiple equilibria.

5.1. **Identifying Labor Supply Parameters.** local labor markets g. We define a firm's inside share, $s_{kj|gt}$, as the firm's employment share of worker type k in year t in labor market g. Following Berry (1994), we derive the following quasi-supply function:

$$\ln \frac{s_{kjt}}{s_{k0t}} = \beta_k \ln \frac{w_{kjt}}{w_{k0t}} + (1 - 1/\sigma_{kg}) \ln s_{kj|gt} + \ln u_{kjt} \tag{5.3}$$

where s_{k0t} and w_{k0t} are the labor market share and earnings of non-employed workers of type k in period t, and u_{kjt} are the unobserved non-pecuniary amenities offered by firm j to workers of type k in year t. We restrict the labor supply parameters to be fixed over time

[26] If there are worker characteristics that influence firms' labor demand that are unobserved by the econometrician, we suggest employing the approach outlined by Bonhomme, Lamadon and Manresa (2019) to estimate these unobserved characteristics. This requires an additional set of assumptions that must be carefully justified before implementation. The specifics of this methodology applied to our setting are provided in Online Appendix D.2.

and across firms, but allow σ_{kg} to vary by worker type and local market, and β_k to vary by labor type.

The parameters of interest are the distribution of unobserved amenities (u_{kjt}) and labor supply elasticities ($\mathcal{E}_{jkt}$) across all firms and worker types. The identification challenge in estimating equation (5.3) is that both the wage and the inside share are potentially correlated with the unobserved amenities and thus endogenous. The most common approach in the industrial organization literature, which we adopt here, is to identify the model parameters using instrumental variables for wages and the inside share. We rewrite our labor quasi-supply function (5.3) in changes as

$$\Delta_{e,e'} \ln \frac{s_{kjt}}{s_{k0t}} = \beta_k \Delta_{e,e'} \ln \frac{w_{kjt}}{w_{k0t}} + \tilde{\sigma}_{kg} \Delta_{e,e'} \ln s_{kj|gt} + \Delta_{e,e'} \ln u_{kjt} \tag{5.4}$$

where $\Delta_{e,e'} x_t \equiv x_{t+e} - x_{t-e'}$ and $\tilde{\sigma}_{kg} \equiv (1 - 1/\sigma_{kg})$. This equation can be consistently estimated using two-stage least squares (2SLS).

For the instruments to be valid, they need to be correlated with long changes ($e + e' + 1$ periods) in the log wage ratio and log inside share (relevance), but orthogonal to long changes in amenities (exogeneity). We follow the IV strategy developed by Lamadon, Mogstad and Setzler (2022) and use internal instruments relying on timing assumptions. Our instruments are short (one-period) changes in log establishment revenue ($\Delta \log R_{jt}$), the log inside share ($\Delta \log s_{kj|gt}$), and the log of the sum of the inside shares for all other labor types employed by the firm ($\Delta \log s_{\sim kj|gt}$). Short changes in these variables will be correlated with long changes in log wages and market shares as long as the labor productivity processes (defined as $\tilde{\gamma}_{kjt}$ in the next section) which determine them are sufficiently persistent.[27] These instruments satisfy the exogeneity assumption as long as the amenity process is sufficiently transitory. Lamadon, Mogstad and Setzler (2022) assume that unobserved firm-specific job amenity shocks are well approximated by a MA(1) process, and show that a choice of $e \geq 2$ and $e' \geq 3$ satisfies the exogeneity assumption. Here, we set $e = 2$, $e' = 3$ and assume that $\text{Cov}(\tilde{\gamma}_{kjt+e} - \tilde{\gamma}_{kjt-e'}, \Delta z_{jkt}) \neq 0$ and $\text{Cov}(\ln u_{kjt+e} - \ln u_{kjt-e'}, \Delta z_{jkt}) = 0$ for each $z_{jkt} \in \{\log R_{jt}, \log s_{kj|gt}, \log s_{\sim kj|gt}\}$, where $\Delta z_{jkt} \equiv z_{jkt} - z_{jkt-1}$. Importantly, this does not restrict correlations between the average *level* of firm-level amenities and labor productivity, nor does it preclude the firm from having chosen the overall level of amenities endogenously.[28] Given the estimated parameters, we can use equation (5.3) to recover amenities ($\ln u_{kjt}$).

[27] In our results, we estimate the labor productivity process as an AR(1) and find that it is highly persistent.

[28] In Online Appendix D.3, we discuss various alternative instrumental variable strategies proposed in the industrial organization, trade, and labor literatures such as shift-share, BLP (using the characteristics of competing firms in the market), and Hausman instruments. We considered these instruments but found that they were not sufficiently strong in our setting. We also implemented a shift-share IV approach following Hummels et al. (2014) and Garin and Silvério (2023). We find labor supply parameter estimates that are comparable to our main estimates despite the fact that we are only able to construct the instrument for the small share of the firms in our sample who export. These results are available upon request.

It is worth noting that, unlike much of the recent literature which seeks to identify labor market power, our approach does not rely directly on the passthrough of firm-specific productivity shocks. The link between passthrough and labor market power is much more complicated in the presence of strategic interactions, as shown in Section 4, and therefore cannot be used to identify the labor supply elasticities.

5.2. **Identifying Labor Demand Parameters.** We assume that the production function for firm j at time t takes the following form:

$$F_t^j(\ell_{\cdot j}) = \left(\sum_{k \in \mathcal{C}_t^j} \tilde{\gamma}_{kjt} \ell_{kjt}^{\rho_k}\right)^{\alpha_{jt}}, \tag{5.5}$$

where $\tilde{\gamma}_{kjt} = \theta_{jt}\gamma_{kjt}$ with $\sum_{k \in \mathcal{C}_t^j} \gamma_{kjt} = 1$ and C_t^j is the set of worker types k employed by firm j in period t. This functional form allows for worker-employer match-specific productivity, whereby a specific type of worker may be more productive in some firms compared to other firms, as in Roy (1951) and, more recently, Lamadon et al. (2022). Our specification generalizes the production function in Lamadon et al. (2022) in two ways. First, we allow worker skill to be multidimensional and do not impose multiplicative separability (i.e., $\gamma_{kj} = \gamma_k \gamma_j$). Second, we relax the assumption that worker types are perfect substitutes.

With this specification, the first-order condition (FOC) in (2.7) becomes

$$\lambda_{jt}\alpha_{jt}\left(\sum_{k \in \mathcal{C}_t^j} \tilde{\gamma}_{kjt} \ell_{kjt}^{\rho_k}\right)^{\alpha_{jt}-1} \tilde{\gamma}_{kjt}\rho_k \ell_{kjt}^{\rho_k - 1} = \frac{\mathcal{E}_{kjt}+1}{\mathcal{E}_{kjt}} w_{kjt} \tag{5.6}$$

Define $\tilde{w}_{kjt} \equiv \frac{\mathcal{E}_{kjt}+1}{\mathcal{E}_{kjt}} w_{kjt}$ (i.e., the marginal revenue product of type k at firm j) and take the ratio of the FOCs for different labor types $k, h \in \mathcal{C}_t^j$ to obtain:

$$\frac{\tilde{\gamma}_{kjt}\rho_k \ell_{kjt}^{\rho_k - 1}}{\tilde{\gamma}_{hjt}\rho_h \ell_{hjt}^{\rho_h - 1}} = \frac{\tilde{w}_{kjt}}{\tilde{w}_{hjt}} \tag{5.7}$$

Taking logs, we have the following log-linear equation:

$$\log \frac{\tilde{w}_{kjt}}{\tilde{w}_{hjt}} = (\rho_k - 1)\log \ell_{kjt} - (\rho_h - 1)\log \ell_{hjt} + \log \frac{\rho_k}{\rho_h} + \log \frac{\tilde{\gamma}_{kjt}}{\tilde{\gamma}_{hjt}}$$

with the last two terms being unobserved by the econometrician. The key parameters of interest are ρ_k, ρ_h, $\tilde{\gamma}_{kjt}$ and $\tilde{\gamma}_{hjt}$.

The identification challenge is that both ℓ_{kjt} and ℓ_{hjt} may be correlated with the unobserved term $\frac{\tilde{\gamma}_{kjt}}{\tilde{\gamma}_{hjt}}$. However, with some assumptions on the structure of $\tilde{\gamma}_{kjt}$ we can obtain internal instruments which allows for consistent estimation of ρ_k and ρ_h. First, we assume that labor productivity for type k, $\tilde{\gamma}_{kjt}$, can be decomposed into an aggregate component $\bar{z}_{kt}$ and a firm-level component z_{jkt} such that $\tilde{\gamma}_{kjt} = \bar{z}_{kt} z_{jkt}$. Second, we assume that the firm-level component follows an AR(1) process in logs: $\log z_{kjt} = \delta_k \log z_{kjt-1} + \bar{\varsigma}_k + \varsigma_{kjt}$ where ς_{kjt} is an i.i.d mean-zero innovation. Finally, we assume that the firm's choice of wages and labor are conditional on $\tilde{\gamma}_{.jt}$ and thus $\varsigma_{.jt}$, but that the innovation is independent from all lagged variables. Substitution leads to the following estimating equation, where we have assumed that $\delta_k = \delta_h \, \forall k, h$:

$$\begin{aligned}\log \frac{\tilde{w}_{kjt}}{\tilde{w}_{hjt}} = c_{kht} + (\rho_k - 1)\log \ell_{kjt} - (\rho_h - 1)\log \ell_{hjt} + \delta \log \frac{\tilde{w}_{kjt-1}}{\tilde{w}_{hjt-1}} \\ - \delta(\rho_k - 1)\log \ell_{kjt-1} + \delta(\rho_h - 1)\log \ell_{hjt-1} + \varsigma_{khjt}\end{aligned} \tag{5.8}$$

where $c_{kht} \equiv \bar{\varsigma}_k - \bar{\varsigma}_h + (1-\delta)\log \frac{\rho_k}{\rho_h} + (\log \bar{z}_{kt} - \log \bar{z}_{ht}) - \delta(\log \bar{z}_{kt-1} - \log \bar{z}_{ht-1})$ is a time-varying constant and $\varsigma_{khjt} \equiv \varsigma_{kjt} - \varsigma_{hjt}$ is i.i.d and mean zero. Note that ℓ_{kjt} and ℓ_{hjt} may be correlated with the error term ς_{khjt}. However, by assumption, ς_{khjt} is uncorrelated with lagged inputs, wages and revenues, allowing us to use functions of these lagged variables as instruments for contemporary input values.[29] This leads to identification of ρ_k, ρ_h and δ.

Estimating equation (5.8) is not straightforward as it is unclear how to choose the (k, h) pairs and construct the instruments/moments for each equation. To deal with this issue, we propose a multi-equation GMM approach which we discuss in detail in Online Appendix D.4.

Given a consistent estimator $\hat{\rho}_k$, we can rearrange the FOC in (5.7) to get

$$\tilde{\gamma}_{hjt} = A_{khjt}\tilde{\gamma}_{kjt}, \quad \text{where} \quad A_{khjt} \equiv \frac{\tilde{w}_{kjt}^{-1}\ell_{kjt}^{\rho_k - 1}\rho_k}{\tilde{w}_{hjt}^{-1}\ell_{hjt}^{\rho_h - 1}\rho_h} \tag{5.9}$$

is a combination of data and known parameters.

Recall that since $\tilde{\gamma}_{kjt} \equiv \theta_{jt}\gamma_{kjt}$ where $\sum_{k \in \mathcal{C}_t^j} \gamma_{kj} = 1$, we have

$$\sum_{h \in \mathcal{C}_t^j \setminus \{k\}} \gamma_{hjt} = \gamma_{kjt} \sum_{h \in \mathcal{C}_t^j \setminus \{k\}} A_{khjt} \Rightarrow (1 - \gamma_{kjt}) = \gamma_{kjt} \sum_{h \in \mathcal{C}_t^j \setminus \{k\}} A_{khjt} \Rightarrow \gamma_{kjt} = \frac{1}{\sum_{h \in \mathcal{C}_t^j} A_{khjt}}$$

for all $k \in \mathcal{C}_t^j$. The first implication holds because $\sum_{k \in \mathcal{C}_t^j} \gamma_{kjt} = 1$, and the second holds because $A_{kkjt} = 1$. This identifies γ_{kjt} for all k, j. Identification of γ_{kjt} and ρ_k does not require any assumptions on the output market. To recover α_{jt} and θ_{jt}, we assume perfect competition in output markets, meaning that each firm j is a price taker on the output market, i.e., $\lambda_{jt} = P_{jt}$ where P_{jt} is the exogenous price. Recall that from equation (5.6), we have:

$$\frac{\tilde{w}_{kjt}}{\gamma_{kjt}\rho_k \ell_{kjt}^{\rho_k - 1}} = \lambda_{jt}\theta_{jt}^{\alpha_j}\alpha_{jt}\left(\sum_{k \in \mathcal{C}_t^j} \gamma_{kjt}\ell_{kjt}^{\rho_k}\right)^{\alpha_{jt}-1}.$$

[29]In the empirical application, we use functions (squares) of lags of the input price ratios and labor input quantities.

By re-arranging and noticing that at the optimum we have $\left(\sum_{k\in\mathcal{C}_t^j}\tilde{\gamma}_{kjt}\ell_{kjt}^{\rho_k}\right)^{\alpha_{jt}} = Q_j$, we obtain the following identification result:

$$\begin{aligned}\alpha_{jt} &= \underbrace{(\lambda_{jt}Q_{jt})}_{R_{jt}}{}^{-1} \times \underbrace{\frac{\mathcal{E}_{kjt}+1}{\mathcal{E}_{kjt}}w_{kjt}}_{\tilde{w}_{kjt}} \times \ell_{kjt} \times \frac{\sum_{k\in\mathcal{C}_t^j}\gamma_{kjt}\ell_{kjt}^{\rho_k}}{\gamma_{kjt}\rho_k\ell_{kjt}^{\rho_k}} \\ &= R_{jt}^{-1} \times \sum_{h\in\mathcal{C}_t^j}\tilde{w}_{hjt}\ell_{hjt}^{1-\rho_h}\rho_h^{-1} \times \sum_{k\in\mathcal{C}_t^j}\gamma_{kjt}\ell_{kjt}^{\rho_k}, \end{aligned} \tag{5.10}$$

where $R_{it} \equiv P_{jt}Q_{jt}$ denotes firm j's total revenue. The second equality holds because of the following:

$$\frac{1}{\gamma_{kjt}} = \tilde{w}_{kjt}^{-1}\ell_{kjt}^{\rho_k-1}\rho_k\sum_{h\in\mathcal{C}_t^j}\frac{1}{\tilde{w}_{hjt}^{-1}\ell_{hjt}^{\rho_h-1}\rho_h} \quad\Rightarrow\quad \frac{\tilde{w}_{kjt}\times\ell_{kjt}}{\gamma_{kjt}\rho_k\ell_{kjt}^{\rho_k}} = \sum_{h\in\mathcal{C}_t^j}\tilde{w}_{hjt}\ell_{hjt}^{1-\rho_h}\rho_h^{-1}.$$

Finally, recall that $R_{it} \equiv P_{jt}Q_{jt} = P_{jt}\theta_{jt}^{\alpha_{jt}}\left(\sum_{k\in\mathcal{C}_t^j}\gamma_{kjt}\ell_{kjt}^{\rho_k}\right)^{\alpha_{jt}}$. Therefore, we obtain $\tilde{\theta}_{jt}$ as

$$\tilde{\theta}_{jt} \equiv P_{jt}^{1/\alpha_{jt}}\theta_{jt} = \frac{R_{jt}^{1/\alpha_{jt}}}{\left(\sum_{k\in\mathcal{C}_t^j}\gamma_{kjt}\ell_{kjt}^{\rho_k}\right)}. \tag{5.11}$$

Note that we could recover θ_{jt} if we observed P_{jt} or normalized P_{jt} to 1.

6. Empirical Application

In this section, we apply our identification strategy to estimate the model parameters using administrative register data from Denmark.

6.1. **Sample Construction and Descriptive Statistics.** We use annual individual and firm registers and the linked employer-employee register IDA (Integrated Database for Labor Market Research) for the years 2001-2019. From the individual register, we get demographic and socio-economic worker characteristics and we identify unemployment and non-employment spells and income. From the firm register, we get yearly value added and revenues arising from the firm's primary operation net of taxes and duties for private-sector firms. The linked employer-employee data contains information on salary, hours/days worked, industry, and workplace location of each employment contract every year. We combine the registers into a yearly panel dataset of workers through unique identifiers for individuals, firms, and establishments. We follow Taber and Vejlin (2020) and Berger et al. (2024) by focusing our empirical analysis on establishments which are linked to a physical location. Establishments are indexed by j and years by t. To get establishment-level revenue R_{jt}, we allocate firm revenue across establishments in proportion to their wage bills. Details on source data, sample selection, and construction of key variables are in Online Appendix E.

We restrict the sample to all individuals between 26 and 60 years of age who work full-time as employees in the private sector and whose job is linked to a physical establishment. We exclude individuals employed in the public and financial sectors due to missing revenue data; financial sector firms are not legally required to report revenue and very few do. In total, our dataset consists of $12,742,741$ individual-year combinations. We assign individuals to 12 observable types k where each type is a combination of gender, age and education.[30]

We measure labor inputs in terms of full-time equivalents (FTE). We use FTEs and worker-establishment linkages to calculate employment variables ℓ_{kjt}, s_{kjt}, and $s_{kj|gt}$ for each worker type k, in establishment j, in year t, overall and by market g. For every k, we also calculate the sum of the inside shares for all other labor types within the establishment, $s_{\sim kj|gt}$. We follow Taber and Vejlin (2020) by using non-employment (unemployment + non-participation) as the outside option. We calculate the share of non-employed workers in the economy every year by worker type k, s_{k0t}, by summing the non-employment spells at the k level and dividing by the total number of FTEs and non-employment spells in the data. The wage w_{ijt} for worker i at establishment j in year t is the total earnings for that worker in the year. We aggregate w_{ijt} to the (k, j, t) level by calculating the mean earnings w_{kjt} for each establishment j and each worker type k, in each year t. We also compute the mean non-employment income w_{k0t} for each worker type in the economy.

We display the 12 k-groups in Table 1, and report descriptive statistics for the full sample of workers in the years 2001-2019. Column 1 reports the share of worker types in the sample, column 2 reports each k-group's average yearly earnings, and column 3 reports the share of establishments employing each k-group. The largest k-group is 36-50-year-old men with lower-than-college education, who make up 24 percent of the sample and are employed by half of the establishments. The smallest group is 51-60-year-old women with a college education, making up only 1.8 percent of the sample and employed by only 5.4 percent of the establishments. The highest earning group is 51-60-year-old men with a college education with average earnings of $106,703$ USD. The lowest earning group is 26-35-year-old women with lower-than-college education with average earnings of $50,775$ USD. The last column of Table 1 shows that the share of establishments employing each k-group is between 5 and 50 percent, reflecting that the number of establishments which are truly available in the labor market for a particular type of worker is lower than the total number of establishments.

For our empirical analysis, we collapse the dataset at the (k, j, t) level leading to $4,487,620$ observations. We further restrict this dataset to only establishments that have no missing values for any of our key variables. These include long and short changes in wages, market

[30]Women represent 31.8 percent of the sample primarily due to women being overrepresented in the Danish public sector (which includes the education and health sectors). The full population of salaried jobs in Denmark in 2001-2019 is 49.3 percent female. This goes down to 35.8 percent when we drop the public sector and to further 31.8 percent when we exclude the financial sector and non-full-time jobs.

	k-group	share of worker-obs.	avg. earnings (in 2022 USD)	share of establishments
1	Female, 26-35, no college	0.046	50,775	0.177
2	Female, 26-35, college	0.033	64,750	0.092
3	Male, 26-35, no college	0.118	61,680	0.365
4	Male, 26-35, college	0.052	77,230	0.137
5	Female, 36-50, no college	0.110	57,347	0.298
6	Female, 36-50, college	0.052	79,674	0.122
7	Male, 36-50, no college	0.238	70,422	0.499
8	Male, 36-50, college	0.095	104,854	0.207
9	Female, 51-60, no college	0.059	56,847	0.192
10	Female, 51-60, college	0.018	77,465	0.054
11	Male, 51-60, no college	0.139	68,621	0.337
12	Male, 51-60, college	0.040	106,703	0.118
Number of worker-observations				12,742,741
Number of unique establishments				259,190

Table 1. Worker distribution across k-groups, all years (2001-2019). Average full-time equivalent (FTE) yearly earnings reported in real-2022 USD. The share of establishments refers to the share of establishments employing each k-group.

shares and inside shares for all other labor types employed by the establishments.[31] Our final dataset contains data for the years 2004-2017 and consists of $1,101,543$ observations at the (k, j, t) level. This selection process leaves us with a subsample of establishments that are larger both in terms of size—each establishment employs on average 11.6 workers instead of the 7.4 in the full sample from 3.5 instead of 2.6 different k-groups—and revenue (8.9 instead of 5.2 million dollars).[32]

We define a local labor market g as a commuting zone and industry pairing. We use the 3-digit industry classification based on NACE Rev. 2 (Carré, 2008) and we drop the public and financial sectors. We use 16 of the 23 commuting zones computed for 2005 by Eckert, Hejlesen and Walsh (2022) using the Tolbert and Sizer (1996) method for Denmark, dropping small islands. In our final estimation dataset, we have 2,757 local labor markets.

Commuting zones and industries (and therefore local markets) vary substantially in the number and type of establishments. The largest commuting zone is Copenhagen, containing around one third of all establishments in Denmark (over $80,000$ unique establishments over the sample period). Copenhagen also contains the largest establishments paying on average the highest wages. In contrast, there are also very small commuting zones with under $2,000$ unique establishments during the time period 2001-2019 (i.e., Ribe and Thisted). In terms of industrial breakdown, the largest industry for number of establishments is wholesale and

[31]For each variable x_{jkt}, we calculate short changes as $x_{jkt} - x_{jkt-1}$, and long changes as $x_{jkt+2} - x_{jkt-3}$, thus restricting the number of years available for the estimation to 2004-2017. Details are available in Online Appendix Table E3.

[32]See Online Appendix Tables F1 and F2 for establishment characteristics for both the full sample and the restricted estimation sample.

retail trade, followed by construction and knowledge-based services. Some industries such as mining, electricity, and water supply are quite small. The local market share $s_{kj|gt}$ is on average 7.4 percent, with significant heterogeneity by commuting zone and industry: it ranges between 1.8 percent in Copenhagen to 29 percent in the more rural areas of Ribe and Thisted, and between 3.2 percent in the construction sector and 31.5 percent in mining and quarrying.

6.2. **Passthrough of Productivity Shocks.** Before proceeding with the estimation of our structural model parameters, we first present suggestive evidence of strategic interactions in the Danish labor market. Specifically, under certain production technologies, e.g., a special case of the production function in equation (5.5) with $\rho_k = 1$, the passthrough of TFP shocks to wages remains constant under monopsonistic competition, but varies with cross-elasticities and market shares under oligopsonistic competition with strategic interactions.

To explore this, we examine whether the passthrough rate depends on an establishment's market share. As reported in Online Appendix D.5, we find that establishments with relatively larger local or national market shares tend to exhibit lower passthrough rates. This pattern aligns with the findings of Morelli and Herkenhoff (2025), who examine passthrough and strategic interactions in credit markets, and is consistent with the presence of strategic interactions at both the local and national levels in Denmark.

6.3. **Estimates of Labor Supply.** We report moments of the distribution of the average labor supply elasticity and markdown estimates in Table 2. The average elasticity across all worker types, establishments, and years is 3.891, and the average markdown is 0.782, meaning that on average wages are marked down 22 percent relative to the marginal revenue product of labor.[33] There is significant heterogeneity in the distribution of labor supply elasticity across establishments and workers, with the 10th and 90th percentiles being 2.244 and 5.629, respectively. Table 2 Panel B shows that the elasticities calculated using the IV-estimated parameters are much larger than the corresponding OLS estimates.[34]

Our estimate of the mean labor supply elasticity is comparable to existing estimates ranging between 3 and 5 (see Card, 2022, and references therein). In particular, Lamadon, Mogstad and Setzler (2022) estimate a labor supply elasticity of 4.2, and Kroft et al. (2025) find estimates ranging between 3.5 and 4.5 for the US construction sector. Berger, Herkenhoff and Mongey (2022) estimate a distribution of firm-specific labor supply elasticities, the

[33]Note that a markdown of 0.782 is slightly lower than what one would obtain by computing the markdown using our average elasticity estimate of 3.891. This is because the markdown is a nonlinear function of the elasticity implying that the average markdown does not equal the ratio of the average elasticity over 1 plus the average elasticity.

[34]Online Appendix Table F3 contains underlying parameter estimates for β_k and σ_{kg} with bootstrapped 95 percent confidence intervals for each k-group. Our IV estimates for β_k are significantly larger than our OLS estimates implying significant downward bias in OLS. The IV estimates for σ_{kg} are slightly smaller than the corresponding OLS estimates.

Panel A. Estimated Labor Supply Parameters					
		Mean	Median	P10	P90
Labor Supply Elasticity (equation (C.1))	$\mathcal{E}_{kjt}$	3.891	3.501	2.244	5.629
Markdown $\left(\text{md}_{kj} = \frac{\mathcal{E}_{kj}}{1+\mathcal{E}_{kj}}\right)$	md_{kjt}	0.782	0.778	0.692	0.849
Cross-wage Super-elasticity (equation (C.3))	ζ_{kjt}	-0.123	-0.034	-0.346	-0.003

Panel B. Elasticities and Markdowns by k-group	IV		OLS	
k-group	$\mathcal{E}_{kjt}$	md_{kjt}	$\mathcal{E}_{kjt}$	md_{kjt}
1. Female, 26-35, no college	5.369	0.843	-0.054	-0.057
2. Female, 26-35, college	7.284	0.879	-0.186	-0.229
3. Male, 26-35, no college	4.632	0.822	0.814	0.449
4. Male, 26-35, college	6.971	0.875	0.631	0.387
5. Female, 36-50, no college	4.212	0.808	0.437	0.304
6. Female, 36-50, college	5.446	0.845	0.107	0.097
7. Male, 36-50, no college	3.265	0.766	0.615	0.381
8. Male, 36-50, college	3.617	0.783	0.182	0.154
9. Female, 51-60, no college	2.681	0.728	0.488	0.328
10. Female, 51-60, college	0.505	0.336	0.310	0.237
11. Male, 51-60, no college	2.726	0.732	0.579	0.367
12. Male, 51-60, college	2.125	0.680	0.292	0.226
Overall	3.501	0.778	0.488	0.328

TABLE 2. Panel A: Moments of the estimated distributions of the establishment- and k-group-level elasticities and markdowns. Panel B: Median of the pooled (over time) distribution of establishment-level labor supply elasticities and markdowns for each k-group, IV and OLS.

average across firms weighted by firm payroll is below 5 and the unweighted average across firms is above 9. The experimental literature finds a wider range of estimates between approximately 2 and 10 (Sokolova and Sorensen, 2021; Bassier, Dube and Naidu, 2022; Emanuel and Harrington, 2025; Dube, Manning and Naidu, 2025). A key feature of our framework is that elasticities vary by worker type, establishment, and market. On the worker side, the overall labor supply elasticity estimate masks significant heterogeneity; the IV estimates in Table 2 Panel B show that median elasticities across worker types range from 0.5 to 7.3. Younger workers tend to have significantly higher elasticities than older workers. Among younger workers, the more educated are more elastic than the less educated but this pattern reverses for older workers. Younger college-educated women and older women have similar or lower elasticity estimates than men, while younger non-college-educated and middle-aged women have higher elasticities than men of the same age and education.[35]

[35] The experimental literature finds on average that women have lower labor supply elasticities than men (Sokolova and Sorensen, 2021), but there are a few exceptions. For example, using experimental evidence from Uber drivers in Houston, Caldwell and Oehlsen (2023) do not find any evidence that firm-specific elasticities differ by gender.

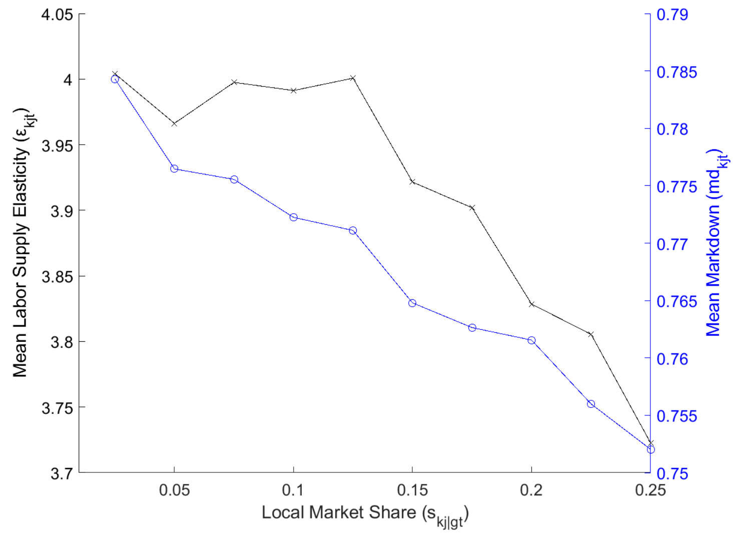


FIGURE 1. Average labor supply elasticity ($\mathcal{E}_{kjt}$) and markdown (md_{kjt}) by local market share ($s_{kj|gt}$). Local market: Commuting Zone×Industry.

On the establishment side, Figure 1 shows that larger establishments—measured according to their inside share—face smaller elasticities and hence mark down wages further below the marginal revenue product of labor. Equation (C.1) in the Online Appendix shows that this relationship is not purely mechanically driven by the Nested Logit functional form since the elasticity also depends on the overall market share and on the k-group- and market-specific variance of the idiosyncratic amenities.[36] We also verify that our empirical estimates satisfy a key requirement for the uniqueness proof of the model equilibrium.

Given the labor supply estimates, we recover the establishment and k-group specific amenity terms u_{kjt} using equation (5.3). In Online Appendix Table F4, we investigate how deterministic preferences for amenities vary across job characteristics. Our results are in line with those for the US reported in Sorkin (2018), who finds evidence of low-value amenities for mining and transportation, as well as a strong contribution of establishment location to amenity values.

6.4. **Estimates of Labor Demand.** We report the production function estimates in Table 3. Panel A reports the average IV and OLS estimates of the labor substitution parameters ρ_k and the persistence of labor productivity δ. The IV estimates for ρ_k are 1.001 on average, range between 0.936 and 1.031, are typically not statistically different from 1, and are fairly similar to the OLS estimates. These estimates imply that the different labor types in our context are highly substitutable, although the actual elasticity of substitution between

[36]Online Appendix Figure F1(b) plots the cross-wage super-elasticity by worker type k as a function of the local market share $s_{kj|gt}$. For very small establishments, this is close to 0 and it declines as establishments get larger.

two labor types at a given establishment will also depend on the relative employment/input levels of these two labor types, as we show below. We find that labor productivity is highly persistent with $\delta = 0.803$.

Panel B reports moments of the distribution of the establishment-level parameters. The distribution of α_{jt} is significantly skewed with mean 0.545 and median 0.194. Similarly, the distribution of the overall productivity term $\tilde{\theta}_{jt}^{\alpha_{jt}}$ is highly skewed: the 90-10 ratio for private sector establishments in Denmark is 3.128.[37] Panel C reports moments of the distribution of the lower bound of passthrough of TFP shocks to wages as derived in Online Appendix C.2, equation (C.4). We find a mean passthrough lower bound of 62 percent.

We next use the production function and labor supply estimates to construct establishment j and k-group specific labor demand elasticities, η_{kjt}, which we report in Panel C. The labor demand elasticities are negative as expected (since higher wages decrease demand for each type of labor). The distribution is fairly skewed, with mean -15.289 and median -5.278, which implies that a 1 percent increase in wage decreases average labor demand by 5.278 percent. There is significant heterogeneity by k-group (see Online Appendix Table F6), with median labor demand elasticities ranging from -3.051 for middle-aged men with no college degree, up to -12.799 for middle-aged women with a college degree.

Recall that the labor productivity parameters γ_{kjt} are normalized at the establishment level. Thus, estimates of γ_{kjt} only have a meaningful interpretation within establishments. To interpret relative differences in labor productivity across k-groups, we regress γ_{kjt} on establishment$\times$year and worker characteristics (Table 3, Panel D). Generally, the estimates show that more educated workers have higher productivity than less educated workers, and that women are less productive than men. We also see that younger workers (age 26-35) are less productive than older workers (age 36+).

To get a better sense of what our production function estimates imply for labor substitutability, we compute the Morishima elasticity of substitution (MEOS, Morishima, 1967). For the standard CES case with two inputs, the MEOS is equivalent to the standard Allen-Uzawa elasticity of substitution. However, when considering non-homogeneous production functions and/or production functions with 3 or more inputs (such as ours), the MEOS represents more accurately the underlying substitution elasticities faced by the firm (Blackorby and Russell, 1989). Moreover, in settings where firms have monopsony power in input markets, it is unclear how to interpret formulations of the elasticity which rely on derivatives with respect to wages (since wages are chosen by firms and are not exogenous) such as the Allen-Uzawa elasticity. Specifically, we use the generalized MEOS derived by Kuga and

[37]This appears high relative to estimated firm productivity ratios in the industrial organization literature; however the measures should not be directly compared, as our model-relevant measure of productivity subsumes both TFP and firm variation in non-labor inputs (capital and intermediates/materials). These estimates are also usually reported within the manufacturing sector, we consider the entire private sector.

Panel A. Estimated Parameters from equation (5.8)					
		IV		OLS	
Persistence of Labor Productivity	δ	0.803 [0.800; 0.805]		0.802 [0.800; 0.804]	
Labor Substitution Parameters (average of ρ_1–ρ_{12})	ρ_k	1.001 [0.987; 1.015]		0.999 [0.995; 1.003]	
Panel B. Distribution of Other Estimated Parameters					
		Mean	Median	P10	P90
Labor Productivity (equation (5.9))	γ_{kjt}	0.287	0.223	0.084	0.543
Scale Parameters (equation (5.10))	α_{jt}	0.545	0.194	0.063	0.459
log of TFP (equation (5.11))	$\log(\tilde{\theta}_{jt}^{\alpha_{jt}})$	8.785	7.437	6.125	9.252
Panel C. Distribution of Labor Demand Elasticities and TFP Passthrough Lower Bound					
		Mean	Median	P10	P90
Labor Demand Elasticities ($\eta_{kj} \equiv F_k^j/\ell_{kj}F_{kk}^j$)	η_{kjt}	-15.289	-5.278	-30.023	-1.431
TFP Passthrough Lower Bound	$\min\left(\frac{\tilde{\theta}_{jt}^{\alpha_{jt}}}{w_{kj}}\frac{\partial w_{kj}}{\partial \tilde{\theta}_{jt}^{\alpha_{jt}}}\right)$	0.617	0.619	0.335	0.909
Panel D. OLS of Labor Productivity (γ_{kjt}) on Worker Characteristics					
Constant (reference: Male, 26-35, no college)		0.210			
College		0.041			
Female		-0.051			
Age 36-50		0.057			
Age 51-60		0.079			

TABLE 3. Parameter estimates for the production function in equation (5.5). Panel A: IV and OLS estimate of δ and the average of our estimates for ρ_k. Bootstrapped 95 percent confidence intervals in square brackets (Hall, 1992) (average of the 12 confidence intervals for ρ_k). Panel B: moments of the estimated distributions of the establishment-level production function parameters (γ_{kjt}, α_{jt}, $\tilde{\theta}_{jt}^{\alpha_{jt}}$). Panel C: moments of the establishment-level labor demand elasticities (η_{kjt}) and the lower bound of the passthrough of TFP shocks to wages (derivation in Online Appendix C.2, equation (C.4)). Panel D: Estimates from OLS of γ_{kjt} on worker characteristics and year×establishment fixed effects (not reported). Reference: Male, 26-35, no college. Number of observations: 2,2660,080. $R^2 = 0.838$. Robust standard errors all below 0.0005, $p < 0.001$. Underlying parameter estimates and full distributions in Online Appendix F.

Murota (1972), where the MEOS of input factor k by h is defined as:

$$MEOS_{khjt} = \frac{F_{ht}^j}{\ell_{kjt}}\frac{H_{khjt}}{H_{jt}} - \frac{F_{ht}^j}{\ell_{hjt}}\frac{H_{hhjt}}{H_{jt}}$$

where $F_{ht}^j = \partial F_t^j/\partial \ell_{hjt}$, ℓ_{kjt} is the level of labor input k, H_{jt} is the bordered Hessian for the production function for establishment j in period t, and H_{khjt} is the cofactor of the $\partial^2 F_t^j/\partial \ell_{kjt}\partial \ell_{hjt}$ term in H. We calculate the MEOS for every input pair, across every establishment, in every period, and report the mean input pair-specific elasticities in Online Appendix Table F7. The estimated elasticities are quite high. Note that the MEOS is not symmetric, unlike the Allen-Uzawa elasticity of substitution. For example, the elasticity of substitution of non-college-educated by college-educated middle-aged men is 23, with the reverse being -19. The pattern of the average MEOS terms broadly follow the estimated ρ_k parameters, with young college-educated women (k-group 2) having both the highest ρ_k parameter, and some of the lowest overall substitution elasticities. Similarly, middle-aged college-educated men (k-group 8) have the lowest ρ_k and highest average elasticities.

6.5. **Sorting.** Using the first-order condition for the optimal wage in equation (5.6), we define the establishment wage premium ($Premium_{jt}$) as the the component of the optimal wage that varies only by establishment j:

$$Premium_{jt} = \alpha_{jt}\tilde{\theta}_{jt}^{\alpha_{jt}} \left(\sum_{k \in \mathcal{C}_t^j} \gamma_{kjt} \ell_{kjt}^{\rho_k} \right)^{\alpha_{jt}-1}$$

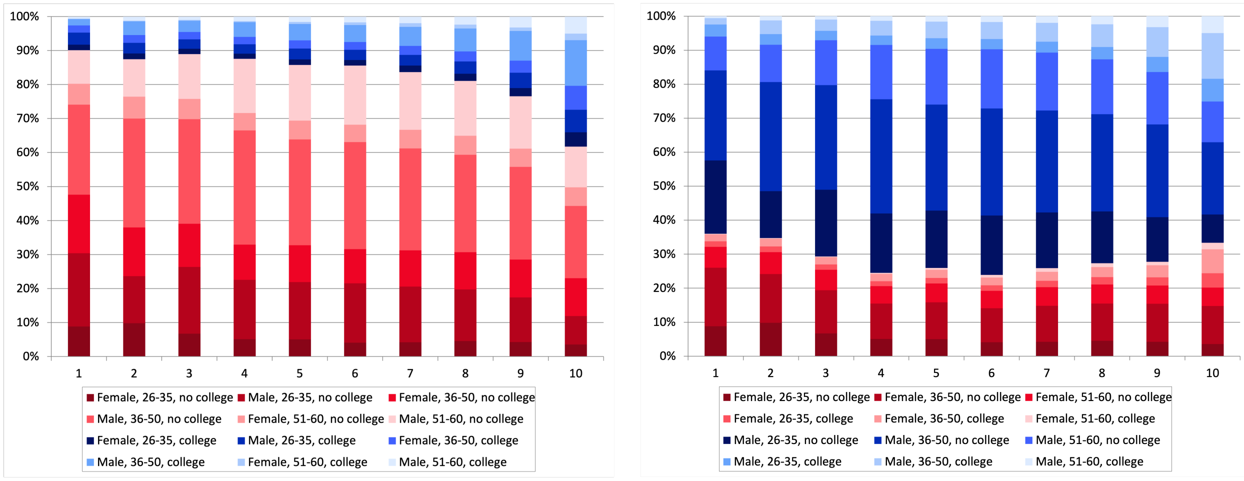
In Figure 2, we illustrate the sorting of worker types across the distribution of the establishment wage premium. The k-groups are ordered slightly differently in panels (a) and (b) to highlight sorting by education and gender, respectively. We find clear evidence of sorting by education: only 10 percent of workers in the bottom decile of the premium distribution are college educated, versus 38 percent in the top decile. We do not find strong evidence of sorting based on gender. If anything, sorting of women appears to follow a U-shaped pattern: women are over-represented in both the bottom and top decile firms. In particular, women tend to sort into higher-premium establishments within sectors with a low share of women such as agriculture, mining, electricity, construction, and transportation, but sort into lower-premium establishments in retail and service sectors.

6.6. **Empirical Analysis of the GCI.** In Section 2.1, we derived the generalized concentration index (GCI) and showed the link between social welfare and labor market concentration. In the Nested Logit Economy, the GCI takes the following form:

$$\begin{aligned} GCI(s_{k\cdot}) \quad &\equiv \quad \exp\left[s_{k0} \ln s_{k0} + \sum_{g=1}^{G} \left[\frac{1}{\sigma_{kg}} \sum_{j \in N_g} s_{kj} \ln s_{kj} + \left(1 - \frac{1}{\sigma_{kg}}\right) s_{kg} \ln s_{kg} \right] \right] \\ &= \quad \Pi_{g=0}^{G} \left(\underbrace{\exp\left\{ \sum_{j \in N_g} s_{kj|g} \ln s_{kj|g} \right\}}_{\text{within-group concentration index}} \right)^{\frac{s_{kg}}{\sigma_{kg}}} \times \underbrace{\exp\left\{ \sum_{g=0}^{G} s_{kg} \ln s_{kg} \right\}}_{\text{between-group concentration index}} \end{aligned} \tag{6.1}$$

where $N_0 \equiv \{0\}$ and $\sigma_{k0} = 1$.

In general, the GCI in a Nested Logit Economy has a very natural and intuitive interpretation. It is a weighted function of "within-group" concentration values, and a "between-group"



(A) sorting over $Premium_{jt}$ by education

(B) sorting over $Premium_{jt}$ by gender

FIGURE 2. Sorting of worker types across deciles of the distribution of the establishment wage premium. This figure shows the employment share of each k-group for each deciles of the establishment-level distribution of wage premium. In Panel (a), the k-groups are ordered by education: non-college graduates in red (older workers in lighter red) and college graduates in blue (older workers in lighter blue). In Panel (a), the k-groups are ordered by gender: women in red (college-educated in lighter red) and men in blue (college-educated in lighter blue). Online Appendix Figure F4 replicates this figure for α_{jt} and $\tilde{\theta}_{jt}^{\alpha_{jt}}$.

component.[38] As pointed out in Maasoumi and Slottje (2003), this type of decomposability of a concentration index is very useful for examining heterogeneity across different local markets. It allows one to identify areas with high concentration levels and the firms that contribute to it. It also allows policy makers to identify the impact of various policy reforms on any desired group of firms and local markets, as well as on overall concentration.

We use our market share data and estimates of σ_{kg} to calculate the GCI for Denmark. Table 4 shows the decomposition of the GCI, as well as k-group-average local market within-group concentration index (WCI) and local and overall market HHI. Column 1 shows the overall GCI which is the product of the WCI aggregated using a weighted geometric mean (column 2) and the between-market index (column 3). The rows of Table 4 are sorted from the most concentrated to least concentrated according to the overall GCI. Non-college-educated women aged 26 to 35 are the group facing the highest market concentration, while college-educated men aged 36 to 50 are the group facing the least concentration.

As a reference point, assume that there are 5 symmetric establishments with equal market share (which is usually interpreted as corresponding a moderate level of concentration). This corresponds to an HHI of 0.2 and a WCI of approximately 0.5. According to this benchmark, roughly 68 percent of local markets have a concentration level above 0.50 when averaging across k-groups (73 percent with the HHI). Moreover, the average level of concentration of

[38]Similarly to the HHI, the different components that form the CGI are special cases to the Hannah and Kay (1977) concentration index.

	k-group	GCI Overall	GCI Within-Group	GCI Between-Group	Local WCI	Local HHI	Overall HHI
1	Female, 26-35, no college	0.0218	0.4528	0.0469	0.7295	0.5181	0.0000
9	Female, 51-60, no college	0.0196	0.4885	0.0390	0.6690	0.4893	0.0001
10	Female, 51-60, college	0.0154	0.4068	0.0362	0.8577	0.6388	0.0002
2	Female, 26-35, college	0.0099	0.4166	0.0233	0.8186	0.6002	0.0003
5	Female, 36-50, no college	0.0060	0.3836	0.0156	0.6144	0.4622	0.0001
6	Female, 36-50, college	0.0040	0.3240	0.0120	0.7561	0.5630	0.0005
11	Male, 51-60, no college	0.0023	0.3579	0.0065	0.5787	0.4406	0.0001
3	Male, 26-35, no college	0.0022	0.3446	0.0065	0.6166	0.4543	0.0001
4	Male, 26-35, college	0.0021	0.2877	0.0073	0.7685	0.5658	0.0005
12	Male, 51-60, college	0.0018	0.2809	0.0064	0.7606	0.5622	0.0003
7	Male, 36-50, no college	0.0013	0.3008	0.0041	0.5362	0.4153	0.0001
8	Male, 36-50, college	0.0010	0.2344	0.0043	0.6913	0.5255	0.0005

Table 4. Columns 1-3: Generalized Concentration Index (GCI) and the contribution of the within- and between-group components as in equation (6.1). Column 1 is the product of columns 2 and 3. Column 4: Within-group Concentration Index (WCI) as in equation (6.1), calculated as the arithmetic mean of the WCI computed for each local market. Columns 5-6: local and overall Herfindahl-Hirschman Index. The local index is calculated as the arithmetic mean of the HHI computed for each local market. The overall HHI is calculated using the whole of Denmark as one market. We rank the k-groups from most concentrated to least concentrated according to the GCI. We calculate the GCI for the full population of private sector establishments in Denmark, extrapolating the σ_{kg} estimates obtained with the restricted estimation sample. All reported numbers are averages over the period 2001-2019.

these concentrated local markets is around 0.94. The WCI also shows significant heterogeneity in concentration by worker type. Local markets for college-educated men tend to be more concentrated than local markets for non-college-educated men in the same age group, but this is the opposite for women. Overall, local markets for women are more concentrated than local markets for men (at all education levels).

Table 4 shows that the low overall GCI (column 1) is driven mainly by low between-market concentration (column 3), while within-market concentration is markedly higher (column 2). In contrast, the overall HHI (column 6) yields a very different ranking across k-groups, reflecting that indices load on different moments of the share distribution: HHI emphasizes second moments, whereas entropy-type measures like the GCI also incorporate higher-order moments (Maasoumi and Theil, 1979). An additional factor that leads to differences between the GCI and the HHI is how it aggregates information across local markets. In particular, equation (6.1) shows that both the overall GCI (column 1) and the within-group GCI (column 2) depend on σ_{kg} which captures the degree of correlation of worker preferences within the local market g. Online Appendix Table F3 shows that low-educated women aged 26 to 35 have a relatively high σ_{kg} estimate. Even though this worker type has below-average concentration levels when ranked according to the HHI (both overall and within-market average, columns $5-6$), it is the most concentrated type when ranked according to the overall GCI, and this difference is driven by the within-group GCI which weights the local entropy index using σ_{kg} (column 2).[39]

7. Counterfactual Analyses

In this section, we use the estimated model to gauge the separate roles of heterogeneity in labor supply, heterogeneity in labor demand, and strategic wage-setting in determining equilibrium wage inequality, concentration, and welfare. To fix ideas, recall that Ξ is the vector of model parameters, and let $\hat{\Xi}$ denote the empirical estimates of these parameters. For example, we denote by $\mathbb{V}^p(\hat{\Xi})$ the variance of log wages predicted by our model. We obtain $\mathbb{V}^p(\hat{\Xi})$ by fixing $\hat{\Xi}$, solving the model equilibrium in equation (3.1) using the Jacobi/Gauss-Seidel algorithm, and computing the variance of log wages in the resulting data. Our counterfactual analysis follows this approach by fixing the model parameters at some counterfactual values, Ξ^c, and then using the model equilibrium to compute the statistics of interest associated with this counterfactual scenario. An important difference in our approach relative to the literature (e.g., Taber and Vejlin, 2020) is that because we always resolve the full equilibrium, every counterfactual encapsulates the general equilibrium reallocation of workers and the resulting feedback to wages and employment.

We consider five scenarios. The first four each eliminate one dimension of heterogeneity, thus serving as a model-based variance decomposition. The final scenario contrasts our oligopsony baseline with a monopsony scenario where firms have fixed markdowns. We list the exact scenarios below, where $\overline{X}$ denotes the employment-weighted mean of X unless stated otherwise:

(1) *Labor supply parameters*:
  - [A]: Remove heterogeneity in deterministic preferences for amenities: $u^c_{kj} = \overline{u}$.
  - [B]: Remove heterogeneity in stochastic preferences for amenities: $\beta^c_k = \overline{\beta}, \sigma^c_{gk} = \overline{\sigma}$.

(2) *Labor demand parameters*:
  - [C]: Remove worker skill heterogeneity within firm j: $\gamma^c_{kj} = \overline{\gamma}$, and no heterogeneity in the rate of substitution across k groups: $\rho^c_k = \overline{\rho}$.
  - [D]: Remove heterogeneity in production technology: $(\tilde{\theta}^{\alpha_j}_j)^c = \overline{\tilde{\theta}^{\alpha}}$, $\alpha^c_j = \overline{\alpha}$.

(3) *Strategic interactions*:
  - [E]: Force every firm to treat the market-level labor supply elasticity as fixed at the "zero-share" elasticity, $\varepsilon^c_{kjt} = \beta_k \sigma_{kg}$.

For each scenario, in Table 5 we report summary statistics for the log wage, elasticities, markdowns, the generalized concentration index (GCI), the non-employment rate, output and welfare. To examine the change in welfare for each scenario, we define the *Equivalent*

[39]The decomposability of the GCI allows us to identify the local markets contributing the most to overall concentration. These are mining and quarrying typically in smaller commuting zones (based on population counts), electricity, gas and steam and water supply/sewage.

Variation, EV_k, as the value which makes workers in group k indifferent between a counterfactual c and the baseline. It is the scalar EV_k that solves $\mathcal{W}_k(W_k \times EV_k, \hat{\Xi}, \lambda, \mathcal{R}) = \mathcal{W}_k(W_k^c, \Xi^c, \lambda, \mathcal{R})$ where $\mathcal{W}_k$ and W_k are the social welfare and vector of wages, respectively, for k-group workers. To attribute welfare changes, we decompose EV_k into two terms following equation (2.13): changes in the deterministic gains from matching, $\Delta m_k \sum_j \tilde{v}_{kj} s_{kj}$ (EV-V), and changes in the GCI, $\Delta(-m_k \ln GCI_k)$ (EV-G). By construction, these contributions sum to $EV_k - 1$. We report the individual-weighted mean Equivalent Variation ($\overline{EV} \equiv \sum_k EV_k \frac{\ell_k}{\sum_k \ell_k}$) and the decomposition in Table 5, with the k-specific EV_k terms reported in Appendix Table F8.[40] Focusing on these key statistics allows us to compare the qualitative and quantitative importance of amenities, worker skills, firm technologies, and strategic interactions in shaping wages and employment. In Table 6, we examine the distributional consequences of strategic interactions.

7.1. **Results.** First, we highlight the key forces in the model that affect wages and how these forces interact. Equation (5.6) shows that wages depend directly on α_{jt}, $\tilde{\gamma}_{kjt}$, ρ_k, the composite term $\left(\sum_{k \in \mathcal{C}_t^j} \tilde{\gamma}_{kjt} \ell_{kjt}^{\rho_k}\right)^{\alpha_{jt}-1}$, and the markdown. The latter is a novel channel through which these primitives can affect wages, since models of monopsonistic competition predict constant markdowns. First, consider the deterministic preference for amenities, u_{kj}. The primary channel through which u_{kj} affect wages is via the labor supplies ℓ_{kjt} in the composite term (and hence the MRPL) and the shares that enter the markdown. Second, consider the stochastic preference for amenities, β_k and σ_{gk}. These enter directly through the labor supply elasticities and hence the markdown and also indirectly through the endogenous labor supplies which enter the MRPL and the shares that enter the markdown. Third, consider worker skill γ_{kj}, the substitution parameter ρ_k, and the production function parameters, $\tilde{\theta}_j^{\alpha_j}$ and α_j. These primitives affect the MRPL directly, and indirectly through the composite term. They also affect markdowns through the endogenous market shares. In general, the effects of shutting down heterogeneity in the model primitives on wages will depend on whether there is heterogeneity in the labor allocation (or worker skill) across firms through the composite term. Since this depends on whether there is heterogeneity in the deterministic preference for amenities (or worker skill), the forces in the model interact.

At a broad level, each factor influences wages through both the markdown and the MRPL, through endogenous labor supplies and market shares. However, an important insight from our counterfactual analyses is that variation in MRPL across firms is the primary driver of wage inequality. This is evident in the first column ("Baseline") of the second panel ("Wage-Variance Decomposition") of Table 5 where, using the first-order condition for wages (equation (5.6)), we find that the total variance of 0.111 breaks down into a 0.139 variance in log MRPL compared to a variance of just 0.019 for log markdowns. The overall wage variance

[40]We do not report social welfare since it is not money-metric and is thus difficult to interpret.

	Baseline	CF A	CF B	CF C	CF D	Monopsony
Wages, Elasticities, and Markdowns						
Mean Log Wage	3.837	3.944	3.879	4.058	3.662	3.859
Mean Labor–Supply Elasticity	3.572	3.963	3.665	3.674	3.659	4.020
Mean Markdown	0.757	0.781	0.781	0.763	0.765	0.780
Wage–Variance Decomposition						
Variance of Log Wages	0.111	0.187	0.180	0.140	0.180	0.111
Variance of Log Markdown	0.019	0.008	0.002	0.017	0.018	0.014
Variance of Log MRPL	0.139	0.180	0.185	0.123	0.177	0.136
$2 \times$ Cov(log Markdown, log MRPL)	−0.047	−0.001	−0.007	−0.000	−0.015	−0.039
Concentration						
GCI	0.008	0.028	0.123	0.003	0.014	0.007
Employment						
Employment rate	0.629	0.576	0.625	0.748	0.519	0.638
Output (bll. DKK)						
Output Y	3,103	3,058	3,142	3,594	1,406	3,109
Welfare						
Equivalent Variation $\overline{EV}$	-	1.520	10.018	1.382	0.547	1.011
EV-V (Matching)	-	0.415	6.113	0.034	-0.240	-0.020
EV-G (Concentration)	-	0.105	2.905	0.348	-0.213	0.031

Table 5. Baseline vs. Counterfactual Scenarios. Means and Variances are *employment-weighted.* GCI values are the across-group means computed in the counterfactual decomposition. MRPL denotes the marginal revenue product of labor. CF A equalizes deterministic amenities ($u_{kj} = \bar{u}$). CF B equalizes idiosyncratic taste–dispersion parameters ($\beta_k = \bar{\beta},\ \sigma_{kg} = \bar{\sigma}$). CF C equalizes worker–firm match productivity and substitution parameters ($\tilde{\gamma}_{kj} = \bar{\gamma},\ \rho_k = \bar{\rho}$). CF D sets firm productivity and returns to scale to the median ($\theta_j^{\alpha_j} = \bar{\theta}^{\bar{\alpha}}$ and $\alpha_j = \bar{\alpha}$). "Monopsony" forces firms to face the elasticity $\varepsilon_{kj} = \beta_k \sigma_{kg}$. The upper bar is a convention to denote that the outcome is an observation-weighted mean, except in D where it represents the median. The mean Equivalent Variation ($\overline{EV}$) is calculated such that welfare in the baseline economy would be the same as a given counterfactual if all baseline workers had their wages multiplied by $\overline{EV}$. We exclude k-group 10 (female, 51-60, college) from this calculation because the estimated wage-preference parameter β for this group is not statistically significant and close to zero, leading to unreliable results. Baseline and counterfactual outcomes are reported for $t = 2015$.

is moderated by the negative covariance between MRPL and markdowns, which reduces the wage variance by 0.047. Of course, this understates the contribution of markdowns to wage inequality; on average, a markdown must exist for firm-level MRPL differences to translate into wage disparities among individuals.

Table 5 displays the main results of our counterfactuals. The first counterfactual labeled "Baseline" is calculated by solving the model using our estimated parameters. Reassuringly, the counterfactual log wage variance evaluated using the estimated parameters (0.111) matches the empirical wage variance in 2015 almost exactly. Thus, the estimated structural model is well suited to investigating the sources of wage inequality in Denmark.

The next set of counterfactual exercises (A–D) highlight the importance of different mechanisms in shaping wage inequality in Denmark. In scenario [A], removing heterogeneity in the deterministic preferences for amenities ($u_{kj} = \bar{u}$) increases wages by 11 percent and the variance of log wages by 68 percent. The mechanism is the positive baseline correlation between firm productivity and amenities: once compensating differentials are eliminated, large high-wage firms raise pay to offset the now less-valued amenities, while smaller low-wage firms reduce pay because amenities there become relatively more valuable. The result is greater dispersion in wages and MRPL, and a higher average wage. On average, workers strongly prefer this scenario: it would take a 52 percent increase in baseline wages to leave them indifferent. As shown in Appendix Table F8, the gains are concentrated among younger workers and women, who are more likely to work at low-amenity firms. Older workers and men respond by exiting employment, lowering the overall employment rate from 63 percent to 58 percent and reducing output by 45 billion DKK. Concentration rises for older workers and, despite a decline for younger workers, this net effect raises overall concentration. By contrast, the welfare gains for younger workers exceed the welfare losses for older workers, producing an aggregate increase in welfare.

In scenario [B], eliminating heterogeneity in the stochastic preferences for amenities across worker types and local markets ($\beta_k = \bar{\beta},\ \sigma_{kg} = \bar{\sigma}$) leads to a rise in wages and wage inequality. In the baseline economy, there is a negative correlation between MRPL and markdowns, which moderates wage disparities across firms. When we remove variation in stochastic preferences, markdowns become nearly uniform, causing the variance in log wages to closely mirror the variance in MRPL, which, in turn, increases from the baseline. As a result, wage inequality increases by 62 percent. This scenario is strongly preferred by older, male, and less-educated workers as these workers have lower preferences for wage income relative to amenities in the baseline scenario, with an average welfare gain of 10. As in scenario A, overall concentration and welfare both rise. Appendix Table F8 indicates this pattern is driven by a composition effect from heterogeneous impacts across age groups. Despite a small decline in employment, output increases by 39 billion DKK as labor reallocates toward higher-wage (higher-MRPL) firms.

Scenario [C] examines the contribution of worker skill ($\tilde{\gamma}_{kj} = \bar{\gamma},\ \rho_k = \bar{\rho}$). Removing heterogeneity in worker skill brings the model closer in structure to that of Berger et al. (2022).[41] We find that eliminating heterogeneity in worker skill increases the variance of log wages by 26 percent. Relative to the baseline economy, as expected, we find that the dispersion (across workers and firms) in the MRPL falls, which acts to reduce wage inequality.

[41]It is important to note that Berger et al. (2022) do not incorporate deterministic preferences for amenities, nor do they allow for heterogeneity in stochastic preferences for amenities across firms and markets. Therefore, scenarios [A] and [B] also shed light on the differences between the modeling approaches.

However, there is a second force in the model which dominates: the covariance across workers and firms between the MRPL and markdown, which is negative in the baseline economy, becomes negligible. This acts to increase overall wage inequality, demonstrating the importance of allowing for heterogeneous markdowns across workers and firms. We also see that concentration decreases which acts to increase welfare overall. Overall employment increases with a corresponding 491 billion DKK increase in output. These changes in concentration, employment, and welfare are driven largely by younger and less-educated workers.

In scenario [D], we fix firm productivity ($\theta_j^{\alpha_j} = \bar{\theta^{\alpha}}$ and $\alpha_j = \bar{\alpha}$) at the median. This mechanically reduces heterogeneity in MRPL. However, in equilibrium, workers endogenously sort into the limited number of firms offering attractive amenities. The variance decomposition shows that this increases dispersion in MRPL and thus the variance of log wages by 62 percent, highlighting the importance of incorporating general equilibrium effects. Equalizing firm productivity depresses mean wages and increases concentration, lowering welfare ($\overline{EV} = 0.547$) since workers in the baseline cluster in more productive firms. Truncating the right tail more than halves output and drives employment down to almost 50 percent.[42]

The final counterfactual ("Monopsony") in the last column of Table 5 evaluates the role of strategic interactions by fixing labor supply elasticities to the "zero-share elasticity" specific to each worker type and labor market ($\varepsilon_{kj} = \beta_k \sigma_{kg}$). Under this assumption, firms treat these elasticities as given and do not internalize how their wage-setting decisions affect the wages offered by other firms in the market. This counterfactual closely resembles the "classic monopsony" framework analyzed in Lamadon et al. (2022). Eliminating strategic interactions raises the average wage by 2.2 percent, primarily due to a 13 percent increase in the average labor supply elasticity and a 3 percent rise in the average markdown. We also find an increase in welfare coming primarily from a decline in concentration. While in principle removing strategic interactions can reduce wage inequality through less dispersion in the markdown across firms, in practice we find that it mainly leads to a shift in the mean wage.

The impact of removing strategic interactions varies across worker types. Table 6 reports the mean log wage, mean elasticity, mean markdown, concentration, the non-employment rate, welfare and a measure of sorting (the mean firm characteristic for each group) for several groups of workers. For each group, the first column shows the baseline outcomes, while the second column reports the percentage changes resulting from the elimination of strategic interactions. We focus on comparing men versus women, and college-educated versus non-college-educated workers. Both men and women experience equal wage gains, driven by a similar increase in markdowns, which leads to higher employment levels. The shift toward monopsony also leads to a reallocation of workers to establishments with higher productivity

[42]Table 5 illustrates that there is no fundamental relationship between concentration and wages. In scenarios [A] and [B], both concentration and wages increase. In scenario [C] and "Monopsony", concentration decreases while wages increase. In scenario [D], concentration increases while wages decrease.

	Female		Male		Non-college		College	
	Base	Mon (%Δ)	Base	Mon (%Δ)	Base	Mon (%Δ)	Base	Mon (%Δ)
Wages, Elasticities, and Markdowns								
Mean Log Wage	3.720	2.3 %	3.884	2.3 %	3.738	2.4 %	4.041	2.0 %
Mean Labor Supply Elasticity	4.111	14.5 %	3.354	11.5 %	3.223	11.6 %	4.293	14.1 %
Mean Markdown	0.769	3.0 %	0.753	3.0 %	0.752	3.2 %	0.769	2.7 %
Concentration								
GCI	0.016	-8.2 %	0.002	-5.0 %	0.008	-7.9 %	0.006	-6.7 %
Employment								
Employment rate	0.476	2.2 %	0.725	1.0 %	0.600	1.5 %	0.703	1.1 %
Welfare								
Equivalent Variation $\overline{EV}_k$	-	1.9 %	-	0.7 %	-	1.4 %	-	0.7 %
EV-V$_k$ (Matching)	-	-2.5 %	-	-2.0 %	-	-2.2 %	-	-2.1%
EV-G$_k$ (Concentration)	-	4.4 %	-	2.7 %	-	3.6 %	-	2.8 %
Sorting								
Mean $log(Premium_{jt})$	6.173	-1.0 %	6.054	-0.8 %	5.934	-0.6 %	6.407	-1.1 %
Mean $alpha_{jt}$	0.235	0.1 %	0.238	0.0 %	0.221	0.2 %	0.271	-0.3 %
Mean $\log(\tilde{\theta}_{jt}^{\alpha_{jt}})$	9.334	1.8 %	9.121	3.5 %	8.993	4.5 %	9.572	1.5 %

Table 6. Baseline vs. Monopsony Counterfactual by Gender and Education. "Mean" rows report *employment-weighted* means, i.e., worker-level means. GCI values are the across-group means computed in the counterfactual decomposition. The "Mon (%Δ)" columns show the percentage change (in levels for log variables) of the monopsony counterfactual relative to the baseline values in the associated "Base" column. The mean Equivalent Variation ($\overline{EV}_k$) is calculated such that welfare in the baseline economy would be the same as a given counterfactual if all baseline workers in that group had their wages multiplied by $\overline{EV}_k$. Baseline and counterfactuals are reported for $t = 2015$.

(especially for men and non-college-educated workers) but lower wage premia (especially women and college-educated workers). Additionally, non-college-educated workers benefit more than their college-educated counterparts, primarily due to a relatively greater wage increase. Part of this comes from a "bargaining effect" where in the new scenario, wages are marked down by less for non-college-educated workers compared to college-educated workers. The remaining part comes from a "selection effect" where non-college-educated workers are relatively more likely to sort to better firms. Although all four groups are better off, women and non-college-educated workers gain three and two times more in terms of $\overline{EV}_k$ than men and college-educated workers, respectively. This is in part due to a relatively larger reduction in concentration for these groups.

8. Conclusion

This paper builds, identifies and estimates a structural two-sided matching model of the labor market featuring strategic interactions. We demonstrate identification of labor supply and demand parameters using instrumental variables and estimate the model parameters using matched employee-employer data from Denmark covering the period 2001-2019. Our empirical results indicate heterogeneity in local markets according to concentration levels and market power of firms which vary both by worker characteristics and firm characteristics. We use our structural model to conduct a series of counterfactual analyses that highlight the roles of heterogeneity and strategic interactions.

Our counterfactuals are a useful first step in demonstrating how our empirical framework can be applied practically to address important questions. Our model and algorithm for solving for the unique equilibrium can be used for examining other applications such as minimum wage reforms, tax and transfer policies, labor market institutions such as unions (as in Dodini, Salvanes and Willén, 2024) and mergers and acquisitions (as in Berger, Herkenhoff and Mongey, 2025).

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Appendix A. Additional results

A.1. **Optimal wage.** Under Assumption 2 (a), the Karush-Kuhn-Tucker (KKT) necessary conditions for optimality of the firm's optimization problem are given by:[43]

(A-1) $\ell_{kj} + w_{kj}\frac{\partial \ell_{kj}}{\partial w_{kj}} - \lambda_j \frac{\partial \ell_{kj}}{\partial w_{kj}} F^j_k(\ell_{\cdot j}) \geq 0,$

(A-2) $w_{kj} \geq 0,$

[43]Notice that in the case where the production functions are non-differentiable (for instance the Leontief Production function) sub-differential versions of KKT conditions are available and can be applied.

(A-3) $w_{kj}\left[\ell_{kj} + w_{kj}\frac{\partial \ell_{kj}}{\partial w_{kj}} - \lambda_j \frac{\partial \ell_{kj}}{\partial w_{kj}} F^j_k(\ell_{\cdot j})\right] = 0$,
(A-4) $F^j(\ell_{\cdot j}) - Y_j \geq 0$,
(A-5) $\lambda_j \geq 0$,
(A-6) $\lambda_j\left[F^j(\ell_{\cdot j}) - Y_j\right] = 0$, for all $(k,j) \in (\mathcal{K} \times \mathcal{J})$.

Given our ARUM and since u_{kj} is finite, $w_{kj} = 0$ implies that $\ell_{kj} = 0$. Under Assumption 2 (b), (A-4) is not violated if there exist some k such $\ell_{kj} > 0$ which under Assumption 1, means $w_{kj} > 0$. This means that each firm in this market pays a strictly positive wage to some types of worker. Let $\mathcal{C}^j \subseteq \mathcal{K}$ denote the set of worker types for whom firm j offers a strictly positive wage. According our ARUM specification and Assumption 1, this is equivalent to $s_{kj} > 0$ and thus, $\mathcal{C}^j \equiv \{k \in \mathcal{K} : s_{kj} > 0\}$. Then, (A-3) implies that (A-1) holds as an equality for all $k \in \mathcal{C}^j$ and thus $\ell_{kj} > 0$ for all $k \in \mathcal{C}^j$. We then have

$$w_{kj} = \lambda_j F^j_k(\ell_{\cdot j})\frac{\mathcal{E}_{kj}}{1+\mathcal{E}_{kj}}, \text{ for all } k \in \mathcal{C}^j. \tag{A.1}$$

Firm j optimally chooses to offer a wage equal to 0 when A-1 holds with strict inequality which corresponds to the case where the marginal cost for this type of worker exceeds the marginal product. Also, notice that all the RHS terms in equation (A.1) have to be positive to ensure that A-4 holds, which is compatible with the previous assumptions used in the model, i.e., Assumption 1, and 2.

Appendix B. Proofs of the main text results

B.1. **Proof of Theorem 1. Fixed point representation of the existence of an equilibrium.** Recall that under Assumptions 1 and 2, the optimal wage (equation (2.7)) can be equivalently written as

$$w_{kj} = \lambda_j F^j_k(\ell_{\cdot j}(w))\frac{\mathcal{E}_{kj}(w)}{1+\mathcal{E}_{kj}(w)} \equiv B_{kj}(w), \quad \forall (k,j) \in \mathcal{K} \times \mathcal{J}. \tag{B.1}$$

where $B(w) \equiv (B_{11}(.), ..., B_{KJ}(.))$. With this representation, showing the existence of an equilibrium matching is equivalent to showing that the mapping $B(w)$ admits at least a fixed point, i.e., w^{eq}, such that $B(w^{eq}) = w^{eq}$ and thus, $s_{kj}(w^{eq}) = \frac{\partial G_{k\cdot}(v_{k\cdot})}{\partial v_{kj}}|_{v_{kj}=v^{eq}_{kj}}$ where $v^{eq}_{kj} \equiv \beta_{kj}\ln w^{eq}_{kj} + \ln u_{kj}$.

Let $\mathbb{T}_0 = \{w : 0 \leq w_{11} \leq \bar{\lambda}\bar{F}', ..., 0 \leq w_{KJ} \leq \bar{\lambda}\bar{F}'\}$, be a closed and bounded rectangular region in $\mathbb{R}^{KJ}$. Consider the iterative procedure defined below, where t indicates a generic iteration step.

Step 0: Let $\underline{\xi}^t = (\underline{\xi}^t_1, ..., \underline{\xi}^t_{I+J})$ and $\overline{\xi}^t = (\overline{\xi}^t_1, ..., \overline{\xi}^t_{I+J})$ be vectors of arbitrarily small non-negative constants such that $\underline{\xi}^t_{kj} \leq w \leq \bar{\lambda}\bar{F}' - \overline{\xi}^t_{kj}$ for all $(k,j) \in \mathcal{K} \times \mathcal{J}$. $\underline{\xi}^t$ is chosen such that some of those components are strictly positive, which is ensured by the fact that under Assumptions 1 and 2, $\mathcal{C}^j \neq \{\emptyset\}$ for each $j \in J$. And define, $\mathbb{T}^t_{\xi} = \{w : \underline{\xi}^t_{11} \leq$

$w_{11} \leq \bar{\lambda}\bar{F}' - \overline{\xi}^t_{11}, ..., \underline{\xi}^t_{KJ} \leq w_{KJ} \leq \bar{\lambda}\bar{F}' - \overline{\xi}^t_{KJ}\}$. Under Assumptions 1 and 2, and given that $B_{kj}(w)$ are continuous functions on a compact set $\mathbb{T}^t_{\xi}$ and $\lambda_j < \bar{\lambda}$, there exist vectors of non-negative constants (some strictly positive) $\underline{\eta}^t = (\underline{\eta}^t_{11}, ..., \underline{\eta}^t_{KJ})$ and $\overline{\eta}^t = (\overline{\eta}^t_{11}, ..., \overline{\eta}^t_{KJ})$ such that $\underline{\eta}^t_{kj} \leq B_{kj}(w) \leq \bar{\lambda}\bar{F}' - \overline{\eta}^t_{kj}$ for all $(k,j) \in \mathcal{K} \times \mathcal{J}$. More precisely, just take $\underline{\eta}^t_{kj} = \inf_{w \in \mathbb{T}^t_{\xi}} B_{kj}(w)$, and $\overline{\eta}^t_{kj} = \bar{\lambda}\bar{F}' - \sup_{w \in \mathbb{T}^t_{\xi}} B_{kj}(w)$, for all $(k,j) \in \mathcal{K} \times \mathcal{J}$.

Step 1: Define $\underline{\xi}^{t+1}_i = \min(\underline{\xi}^t_i, \underline{\eta}^t_i)$ for for $i = 11, ..., KJ$ and $\overline{\xi}^{t+1}_i = \min(\overline{\xi}^t_i, \overline{\eta}^t_i)$ for $i = 11, ..., KJ$.

Step 2: If $\underline{\xi}^{t+1}_i = \underline{\xi}^t_i$ and $\overline{\xi}^{t+1}_i = \overline{\xi}^t_i$ then stop and define $\underline{\epsilon}_i = \underline{\xi}^{t+1}_i$, $\overline{\epsilon}_i = \overline{\xi}^{t+1}_i$.

Step 3: If $\underline{\xi}^{t+1}_i \neq \underline{\xi}^t_i$ or $\overline{\xi}^{t+1}_i \neq \overline{\xi}^t_i$ then $t \leftarrow t+1$ and go back to step 0. percent= $\underline{\xi}^t_i$ and $\overline{\xi}^{t+1}_i = \overline{\xi}^t_i$ then t go to the last step.

By construction, the sequences $\underline{\xi}^t_i$ and $\overline{\xi}^t_i$ are positive, decreasing, and bounded below by 0, and therefore both sequences converge. Therefore, when the iteration stops in **Step 2**, let $\mathbb{T}_{\epsilon} = \{w : \underline{\epsilon}_{11} \leq w_{11} \leq \bar{\lambda}\bar{F}' - \overline{\epsilon}_{11}, ..., \underline{\epsilon}_{KJ} \leq w_{KJ} \leq \bar{\lambda}\bar{F}' - \overline{\epsilon}_{KJ}\}$ be a closed and bounded rectangular region in $\mathbb{R}^{KJ}$.

$B(w)$ is a continuously differentiable mapping such that $B(w)$: $\mathbb{T}_{\epsilon} \to \mathbb{T}_{\epsilon}$. Thus, the existence of a wage equilibrium w^{eq} is ensured by invoking the Brouwer fixed-point theorem. Then, by construction we have the existence of (s^{eq}, w^{eq}).

B.2. **Proof of Theorem 2.** Let's define

$$\delta_{kj}(w) \equiv w_{kj} - \lambda_j F^j_k(\ell_{\cdot j}(w)) \frac{\mathcal{E}_{kj}(w)}{1 + \mathcal{E}_{kj}(w)}, \quad \forall (k,j) \in \mathcal{K} \times \mathcal{J}. \tag{B.2}$$

where $\delta(w) = (\delta_{11}(w), ..., \delta_{KJ}(w)) : \mathbb{T}_{\epsilon} \subseteq \mathbb{R}^{KJ} \longrightarrow \mathbb{R}^{KJ}$. Given existence of an equilibrium matching from Theorem 1, showing uniqueness is equivalent to show the global univalence of the mapping $\delta(w)$. Under Assumptions 1 and 2, $\delta(w)$ is continuously differentiable. Let $\mathbb{J}_{\delta}(w)$ be its Jacobian matrix,

$$\underset{(KJ \times KJ)}{\mathbb{J}_{\delta}(w)} = \begin{pmatrix} \frac{\partial \delta_{11}}{\partial w_{11}} & \cdots & \frac{\partial \delta_{11}}{\partial w_{KJ}} \\ \vdots & \ddots & \vdots \\ \frac{\partial \delta_{KJ}}{\partial w_{11}} & \cdots & \frac{\partial \delta_{KJ}}{\partial w_{KJ}} \end{pmatrix}$$

According to Gale and Nikaido (1965), we know that $\delta(w)$ is globally univalent on $\mathbb{T}_{\epsilon}$ if $\mathbb{J}_{\delta}(w)$ is a P-matrix for all $w \in \mathbb{T}_{\epsilon}$. In the rest of the proof, we show that $\mathbb{J}_{\delta}(w)$ is indeed a P-matrix whenever Assumption 3 holds.

In the following we will make use extensive use of the following lemma:

Lemma 1. *Under Assumption 1, the following shape restrictions hold:*

$$\frac{\partial s_{kj}}{\partial w_{kl}} \begin{cases} \geq 0, & \text{if } l = j \\ \leq 0, & \text{if } l \in \mathcal{J}_0 \setminus \{j\} \end{cases}$$

Proof.

$$\begin{aligned} s_{kj} &= \mathbb{P}\big(v_{kj} + \epsilon_{ij} \geq v_{kj'} + \epsilon_{ij'} \text{ for all } j' \in \mathcal{J} \cup \{0\} \equiv \mathcal{J}_0\big) \\ &= \mathbb{P}\left(\underbrace{\epsilon_{i0} - \epsilon_{ij}}_{\varepsilon_{ij0}} \leq v_{kj} - v_{k0}, ..., \underbrace{\epsilon_{iJ} - \epsilon_{ij}}_{\varepsilon_{ijJ}} \leq v_{kj} - v_{kJ}\right) \\ &= F_{\varepsilon_{ij0},...,\varepsilon_{ijJ}}(v_{kj} - v_{k0}, ..., v_{kj} - v_{kJ}). \end{aligned}$$

Let $F^{(l)}_{X1,...,X_J}(x_1, ..., x_J) \equiv \frac{\partial}{\partial x_l} F_{X1,...,X_J}(x_1, ..., x_J)$. Then, we have:

$$\begin{aligned} \frac{\partial s_{kj}}{\partial v_{kl}} &= -F^{(l)}_{\varepsilon_{ij0},...,\varepsilon_{ijJ}}(v_{kj} - v_{k0}, ..., v_{kj} - v_{kJ}) \leq 0, \text{ for } l \neq j, \\ \frac{\partial s_{kj}}{\partial v_{kj}} &= \sum_{l \neq j} F^{(l)}_{\varepsilon_{ij0},...,\varepsilon_{ijJ}}(v_{kj} - v_{k0}, ..., v_{kj} - v_{kJ}) \geq 0, \end{aligned}$$

where both inequalities hold because $F_{\varepsilon_{ij0},...,\varepsilon_{ijJ}}(.)$ is a joint CDF. ∎

Definition 2. *Let A be a real square matrix. (i) A is a P-matrix if every principal minor of A is positive, i.e., > 0. (ii) A is said to be a* **positive diagonally dominant** *matrix if there exists a strictly positive vector $d = (d_1, ..., d_n)$ where each $d_i > 0$ such that $d_i A_{ii} > \sum_{j \neq i} d_j |A_{ij}|$.*

According to Proposition 1(ii) in Parthasarathy (2006), any real square matrix that is positive diagonally dominant is a P-matrix. Recall that under Assumption 2, $\mathcal{C}^j \neq \{\emptyset\}$. In fact, in our modeling approach, $\lambda_j F^j_k(\ell_{\cdot j}(w)) \frac{\mathcal{E}_{kj}(w)}{1+\mathcal{E}_{kj}(w)} = 0 \iff F^j_k(\ell_{\cdot j}(w)) = 0$ for all $w \in \mathbb{T}_\epsilon$, but according to Assumption 2, for each $j \in \mathcal{J}$ there exists at least some k such that $F^j_k(\ell_{\cdot j}(w)) > 0$ then $\lambda_j F^j_k(\ell_{\cdot j}(w)) \frac{\mathcal{E}_{kj}(w)}{1+\mathcal{E}_{kj}(w)} > 0$. Under Assumptions 1 and 2, for all $k \in \mathcal{C}^j$ and $j \in \mathcal{J}$, we have

$$\frac{\partial \delta_{kj}}{\partial w_{ml}} = \begin{cases} 1 - \lambda_j \frac{\partial \ell_{kj}(w_{k\cdot})}{\partial w_{kj}} F^j_{kk}(\ell_{\cdot j}(w)) \frac{\mathcal{E}_{kj}(w_{k\cdot})}{1+\mathcal{E}_{kj}(w_{k\cdot})} - \lambda_j F^j_k(\ell_{\cdot j}(w)) \frac{1}{(1+\mathcal{E}_{kj}(w_{k\cdot}))^2} \frac{\partial \mathcal{E}_{kj}(w_{k\cdot})}{\partial w_{kj}}, & \text{if } m = k, l = j \\ -\lambda_j \frac{\partial \ell_{kj}(w_{k\cdot})}{\partial w_{kl}} F^j_{kk}(\ell_{\cdot j}(w)) \frac{\mathcal{E}_{kj}(w_{k\cdot})}{1+\mathcal{E}_{kj}(w_{k\cdot})} - \lambda_j F^j_k(\ell_{\cdot j}(w)) \frac{1}{(1+\mathcal{E}_{kj}(w_{k\cdot}))^2} \frac{\partial \mathcal{E}_{kj}(w_{k\cdot})}{\partial w_{kl}}, & \text{if } m = k, l \neq j \\ -\lambda_j \frac{\partial \ell_{mj}(w_{m\cdot})}{\partial w_{ml}} F^j_{km}(\ell_{\cdot j}(w)) \frac{\mathcal{E}_{kj}(w_{k\cdot})}{1+\mathcal{E}_{kj}(w_{k\cdot})}, & \text{if } m \neq k. \end{cases}$$

for all $(m, l) \in \mathcal{K} \times \mathcal{J}$. Notice that for all $k \in \overline{\mathcal{C}^j} \equiv \mathcal{K} \setminus \mathcal{C}^j, j \in \mathcal{J}$, because $F^j_k(\ell_{\cdot j}(w)) = 0$ we have $\frac{\partial \delta_{kj}}{\partial w_{kj}} = 1$ and $\frac{\partial \delta_{kj}}{\partial w_{ml}} = 0$ for $m \neq k$ or $l \neq j$. For all $k \in \mathcal{C}^j$ denote $d_{kj} \equiv w_{kj}/\beta_{kj} > 0$ and for all $k \in \overline{\mathcal{C}^j}$ $d_{kj} = 1$ and this for all $j \in \mathcal{J}$. Consider two cases:

Case 1: Assumption 3 holds: Under Assumption 3 we have the following sign restriction on $\frac{\partial \delta_{kj}}{\partial w_{ml}}$:

$$\frac{\partial \delta_{kj}}{\partial w_{ml}} = \begin{cases} 1 - \lambda_j \underbrace{\frac{\partial \ell_{kj}(w_{k\cdot})}{\partial w_{kj}}}_{\geq 0} \underbrace{F^j_{kk}(\ell_{\cdot j}(w))}_{\leq 0} \frac{\mathcal{E}_{kj}(w_{k\cdot})}{1+\mathcal{E}_{kj}(w_{k\cdot})} - \lambda_j F^j_k(\ell_{\cdot j}(w)) \frac{1}{(1+\mathcal{E}_{kj}(w_{k\cdot}))^2} \underbrace{\frac{\partial \mathcal{E}_{kj}(w_{k\cdot})}{\partial w_{kj}}}_{\leq 0} > 0, \text{ if } m = k, l = j \\ -\lambda_j \underbrace{\frac{\partial \ell_{kj}(w_{k\cdot})}{\partial w_{kl}}}_{\leq 0} \underbrace{F^j_{kk}(\ell_{\cdot j}(w))}_{\leq 0} \frac{\mathcal{E}_{kj}(w_{k\cdot})}{1+\mathcal{E}_{kj}(w_{k\cdot})} - \lambda_j F^j_k(\ell_{\cdot j}(w)) \frac{1}{(1+\mathcal{E}_{kj}(w_{k\cdot}))^2} \underbrace{\frac{\partial \mathcal{E}_{kj}(w_{k\cdot})}{\partial w_{kl}}}_{\geq 0} \leq 0, \text{ if } m = k, l \neq j, \\ -\lambda_j \frac{\partial \ell_{mj}(w_{m\cdot})}{\partial w_{ml}} \underbrace{F^j_{km}(\ell_{\cdot j}(w))}_{=0} \frac{\mathcal{E}_{kj}(w_{k\cdot})}{1+\mathcal{E}_{kj}(w_{k\cdot})} = 0, \text{ if } m \neq k. \end{cases}$$

Therefore, for all $k \in \mathcal{C}^j$ and $j \in \mathcal{J}$, we can show that

$$\begin{aligned} &\frac{w_{kj}}{\beta_{kj}} \frac{\partial \delta_{kj}}{\partial w_{kj}} - \sum_{m \neq k \text{ or } l \neq j} \frac{w_{ml}}{\beta_{ml}} \left| \frac{\partial \delta_{kj}}{\partial w_{ml}} \right| = \\ &\quad \underbrace{\frac{w_{kj}}{\beta_{kj}}}_{>0} - \lambda_j \underbrace{\left[\frac{w_{kj}}{\beta_{kj}} \frac{\partial \ell_{kj}(w_{k\cdot})}{\partial w_{kj}} + \sum_{l \neq j} \frac{w_{kl}}{\beta_{kl}} \frac{\partial \ell_{kj}(w_{k\cdot})}{\partial w_{kl}} \right]}_{m_k \sum_{l \in \mathcal{J}} \frac{\partial s_{kj}(w_{k\cdot})}{\partial v_{kl}} = -m_k \frac{\partial s_{kj}(w_{k\cdot})}{\partial v_{k0}} \geq 0} \underbrace{F^j_{kk}(\ell_{\cdot j}(w))}_{\leq 0} \frac{\mathcal{E}_{kj}(w_{k\cdot})}{1+\mathcal{E}_{kj}(w_{k\cdot})} \\ &\quad - \lambda_j \underbrace{F^j_k(\ell_{\cdot j}(w))}_{\geq 0} \frac{1}{(1+\mathcal{E}_{kj}(w_{k\cdot}))^2} \underbrace{\left[\frac{w_{kj}}{\beta_{kj}} \frac{\partial \mathcal{E}_{kj}}{\partial w_{kj}} + \sum_{l \neq j} \frac{w_{kl}}{\beta_{kl}} \frac{\partial \mathcal{E}_{kj}(w_{k\cdot})}{\partial w_{kl}} \right]}_{\sum_{l \in \mathcal{J}} \frac{\partial \mathcal{E}_{kj}(w_{k\cdot})}{\partial v_{kl}} = -\frac{\partial \mathcal{E}_{kj}}{\partial v_{k0}} \leq 0} > 0. \end{aligned} \tag{B.3}$$

All the sign restrictions hold under Assumption 3. Two main non-obvious points in the previous inequality are the following equalities: $\sum_{l \in \mathcal{J}_0} \frac{\partial s_{kj}(w_{k\cdot})}{\partial v_{kl}} = 0$ and $\sum_{l \in \mathcal{J}_0} \frac{\partial \mathcal{E}_{kj}(w_{k\cdot})}{\partial v_{kl}} = 0$. The trick behind these equalities is the fact that an increase of all mean gross utility $v_{k\cdot}$ does not affect the share s_{kj}, as remarked by Berry (1994), and the elasticity $\mathcal{E}_{kj}$. Moreover, for all $k \in \overline{\mathcal{C}^j}$, and $j \in \mathcal{J}$, $d_{kj} \frac{\partial \delta_{kj}}{\partial w_{kj}} - \sum_{m \neq k \text{ or } l \neq j} d_{ml} \left| \frac{\partial \delta_{kj}}{\partial w_{ml}} \right| > 0$ trivially holds. Therefore, $\mathbb{J}_\delta(w)$ is indeed a P-matrix for all $w \in \mathbb{T}_\epsilon$, and then $\delta(w)$ is globally univalent on $\mathbb{T}_\epsilon$, which completes the proof.

Case 2: Assumption 3 (i) holds: In such a context we can show that

$$\frac{w_{kj}}{\beta_{kj}}\frac{\partial \delta_{kj}}{\partial w_{kj}} - \sum_{m\neq k \text{ or } l\neq j} \frac{w_{ml}}{\beta_{ml}}\left|\frac{\partial \delta_{kj}}{\partial w_{ml}}\right| =$$

$$\frac{w_{kj}}{\beta_{kj}} + \lambda_j \sum_{m\neq k} \underbrace{\left[-\frac{w_{mj}}{\beta_{mj}}\frac{\partial \ell_{mj}(w_{m\cdot})}{\partial w_{mj}} + \sum_{l\neq j}\frac{w_{ml}}{\beta_{ml}}\frac{\partial \ell_{mj}(w_{m\cdot})}{\partial w_{ml}}\right]}_{-\frac{\partial \ell_{mj}}{\partial v_{m0}} - 2\frac{\partial \ell_{mj}}{\partial v_{mj}}} \left|F^j_{km}(\ell_{\cdot j}(w))\right| \frac{\mathcal{E}_{kj}(w_{k\cdot})}{1+\mathcal{E}_{kj}(w_{k\cdot})}$$

$$-\lambda_j \underbrace{\left[\frac{w_{kj}}{\beta_{kj}}\frac{\partial \ell_{kj}(w_{k\cdot})}{\partial w_{kj}} + \sum_{l\neq j}\frac{w_{kl}}{\beta_{kl}}\frac{\partial \ell_{kj}(w_{k\cdot})}{\partial w_{kl}}\right]}_{m_k \sum_{l\in\mathcal{J}} \frac{\partial s_{kj}(w_{k\cdot})}{\partial v_{kl}} = -m_k \frac{\partial s_{kj}(w_{k\cdot})}{\partial v_{k0}} \geq 0} \underbrace{F^j_{kk}(\ell_{\cdot j}(w))}_{\leq 0} \frac{\mathcal{E}_{kj}(w_{k\cdot})}{1+\mathcal{E}_{kj}(w_{k\cdot})}$$

$$-\lambda_j \underbrace{F^j_{k}(\ell_{\cdot j}(w))}_{\geq 0} \frac{1}{(1+\mathcal{E}_{kj}(w_{k\cdot}))^2} \underbrace{\left[\frac{w_{kj}}{\beta_{kj}}\frac{\partial \mathcal{E}_{kj}}{\partial w_{kj}} + \sum_{l\neq j}\frac{w_{kl}}{\beta_{kl}}\frac{\partial \mathcal{E}_{kj}(w_{k\cdot})}{\partial w_{kl}}\right]}_{\sum_{l\in\mathcal{J}} \frac{\partial \mathcal{E}_{kj}(w_{k\cdot})}{\partial v_{kl}} = -\frac{\partial \mathcal{E}_{kj}}{\partial v_{k0}} \leq 0}.$$

Notice that the second term after the equality holds because, as discussed earlier, we have $\sum_{l\in\mathcal{J}} \frac{\partial s_{mj}(w_{m\cdot})}{\partial v_{ml}} = -\frac{\partial s_{mj}(w_{m\cdot})}{\partial v_{m0}}$. Therefore, we can write:

$$\frac{w_{kj}}{\beta_{kj}}\frac{\partial \delta_{kj}}{\partial w_{kj}} - \sum_{m\neq k \text{ or } l\neq j} \frac{w_{ml}}{\beta_{ml}}\left|\frac{\partial \delta_{kj}}{\partial w_{ml}}\right| = \frac{w_{kj}}{\beta_{kj}} + \lambda_j \left\{ -\sum_{m\neq k}\left[\frac{\partial \ell_{mj}(w_{m\cdot})}{\partial v_{m0}} + 2\frac{\partial \ell_{mj}(w_{m\cdot})}{\partial v_{mj}}\right]\left|F^j_{km}(\ell_{\cdot j}(w))\right| \right.$$
$$\left. + \frac{\partial \ell_{kj}(w_{k\cdot})}{\partial v_{k0}} F^j_{kk}(\ell_{\cdot j}(w)) + F^j_k(\ell_{\cdot j}(w)) \frac{1}{(1+\mathcal{E}_{kj}(w_{k\cdot}))\mathcal{E}_{kj}(w_{k\cdot})}\frac{\partial \mathcal{E}_{kj}}{\partial v_{k0}} \right\} \times \frac{\mathcal{E}_{kj}(w_{k\cdot})}{1+\mathcal{E}_{kj}(w_{k\cdot})}.$$

Without additive separability in the production function the equilibrium can be unique if the RHS of the latter equality is positive. A sufficient condition for it is that

$$\left\{ -\sum_{m\neq k}\left[\frac{\partial \ell_{mj}(w_{m\cdot})}{\partial v_{m0}} + 2\frac{\partial \ell_{mj}(w_{m\cdot})}{\partial v_{mj}}\right]\left|F^j_{km}(\ell_{\cdot j}(w))\right| + \frac{\partial \ell_{kj}(w_{k\cdot})}{\partial v_{k0}} F^j_{kk}(\ell_{\cdot j}(w)) \right.$$
$$\left. + F^j_k(\ell_{\cdot j}(w)) \frac{1}{(1+\mathcal{E}_{kj}(w_{k\cdot}))\mathcal{E}_{kj}(w_{k\cdot})}\frac{\partial \mathcal{E}_{kj}}{\partial v_{k0}} \right\} \geq 0$$

for all $w \in \mathbb{T}_\epsilon$.

B.3. **Proof of Proposition 1.**

Lemma 2. *Under Assumptions 1, 2, and 3, $\delta(w)$ is generalized nonlinear diagonally dominant on $\mathbb{T}_\epsilon$.*

Proof. All partial derivatives of $\delta(w)$ exist and are continuous. Let's $\mathbb{J}_\delta(w) \equiv \delta(w)'$ be its Jacobian matrix which is continuous on $\mathbb{T}_\epsilon$. $\delta(w)$ is Frèchet-differentiable on $\mathbb{T}_\epsilon$ then it is Gâteaux-differentiable on $\mathbb{T}_\epsilon$ which is a convex compact subset of $\mathbb{R}_{KJ}$. In case 1 of the Proof of Theorem 2, we show that $\mathbb{J}_\delta(w)$ is *a generalized diagonally dominant matrix* in the language of Gan, Huang and Gao (2006) for all $w \in \mathbb{T}_\epsilon$. The proof is complete once we invoke Theorem 8 of Gan, Huang and Gao (2006). $\square$

Lemma 3. *Under Assumptions 1, 2, and 3, for any $w \in \mathbb{T}_\epsilon$, and $kj = 1, ..., KJ$ the following equation in x_{kj}: $\psi(x_{kj}, w_{-kj}) \equiv \delta_{kj}(w_{11}, ..., w_{1J}, ..., w_{k,j-1}, x_{kj}, w_{k,j+1}, ..., w_{KJ}) = 0$ as a unique solution x^*_{kj}.*

Proof. In the case 1 of the Proof of Theorem 2, we show that $\frac{\partial \psi(x_{kj}, w_{-kj})}{\partial x_{ij}} \geq 1 > 0$, then $\psi(x_{kj}, w_{-kj})$ is strictly increasing in x_{kj} for any $w_{-kj} \in \mathbb{T}_\epsilon$. In addition, as can be seen in the proof of Theorem 1, $\psi(\underline{\epsilon}_{kj}, w_{-kj}) \leq 0 \leq \psi(\bar{\lambda}\bar{F}' - \bar{\epsilon}_{kj}, w_{-kj})$ for any $w_{-kj} \in \mathbb{T}_\epsilon$. This completes the proof. $\square$

Under Assumptions 1, 2, and 3, and if Lemmata 2, and 3 hold, we can invoke Theorem 18 in Frommer (1991). Remark that both underrelaxed Gauss-siedel and Jacobi iteration are special cases of the asynchronous iterative methods discussed in Frommer (1991) Theorem 18. This completes the proof of Proposition 1.

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ONLINE APPENDIX
An Empirical Framework for Matching with Imperfect Competition.

Mons Chan[1], Kory Kroft[2], Elena Mattana[3], Ismael Mourifié[4]

Appendix C. Nested Logit Economy.

C.1. **Elasticities, Cross-wage super-elasticities, Equilibrium Uniqueness.** To allow unobserved workers preferences ϵ_{ij} to be correlated for certain classes of firms, we partition the J firms into G nests, where each nest is a local labor market. The g^{th} nest contains N_g firms. We assume the ϵ_{ij} to be correlated within nests, i.e., $1/\sigma_{kg} = \sqrt{1 - corr(\epsilon_{ij}, \epsilon_{il})}$ for $j \neq l$ where for $(j,l) \in N_g$, and with $\sigma_{kg} \in [1, \infty)$. Despite the nesting structure, we allow each firm to compete with every other firm in the economy, regardless of whether firms belong to the same nest or not.

In this Nested Logit Economy, the social surplus function is given by

$$G_{k\cdot}(v_{k\cdot}) = \ln \Big\{ \overbrace{e^{v_{k0}} + \sum_{g=1}^{G} \Big(\underbrace{\sum_{j \in N_g} e^{v_{kj}\sigma_{kg}}}_{\mathcal{I}_{k,g}(v_{k\cdot})} \Big)^{1/\sigma_{kg}}}^{\mathcal{I}_{k,M}(v_{k\cdot})} \Big\},$$

where $\mathcal{I}_{k,g}(v_{k\cdot})$ and $\mathcal{I}_{k,M}(v_{k\cdot})$ denote, respectively, the aggregate weighted wage index at the local market g level, and at the "national" market level. Additionally, the market shares have the following weakly separable functional form: $s_{kj}(w_{k\cdot}) = f\big(w_{kj}, \mathcal{I}_{k,g}(v_{k\cdot}), \mathcal{I}_{k,M}(v_{k\cdot})\big)$. The labor supply elasticities are given by:

$$\begin{aligned}\mathcal{E}_{kj} = \frac{w_{kj}}{s_{kj}} \Bigg[f_1\big(w_{kj}, \mathcal{I}_{k,g}(v_{k\cdot}), \mathcal{I}_{k,M}(v_{k\cdot})\big) \quad &+ \quad \frac{\partial \mathcal{I}_{k,g}(v_{k\cdot})}{\partial w_{kj}} f_2\big(w_{kj}, \mathcal{I}_{k,g}(v_{k\cdot}), \mathcal{I}_{k,M}(v_{k\cdot})\big) \\ &+ \quad \frac{\partial \mathcal{I}_{k,M}(v_{k\cdot})}{\partial w_{kj}} f_3\big(w_{kj}, \mathcal{I}_{k,g}(v_{k\cdot}), \mathcal{I}_{k,M}(v_{k\cdot})\big) \Bigg]\end{aligned}$$

where $f_k(x_1, x_2, x_3) = \frac{\partial f(x_1,x_2,x_3)}{\partial x_k}$ for $k \in \{1, 2, 3\}$. The last equality shows that a change in w_{kj} has a direct effect on the share s_{kj} captured by $f_1(.)$ and two indirect effects mediated

[1]Department of Economics, Queen's University, mons.chan@queensu.ca.
[2]Department of Economics, University of Toronto, & NBER, kory.kroft@utoronto.ca.
[3]Department of Economics and Business Economics, Aarhus University,, emattana@econ.au.dk.
[4]Department of Economics, Washington University in St. Louis & NBER, ismaelm@wustl.edu.

by the impact of the change of w_{kj} on the local and the total market indexes $\mathcal{I}_{k,g}(v_{k\cdot})$, and $\mathcal{I}_{k,M}(v_{k\cdot})$, respectively.

The elasticity of labor supply in the Nested Logit economy takes the following form:

$$\mathcal{E}_{kj} \quad = \quad \beta_{kj}[\sigma_{kg} + (1-\sigma_{kg})s_{kj|g} - s_{kj}] \quad \text{for } j \in N_g \tag{C.1}$$

with $s_{kj} \equiv e^{v_{kj}\sigma_{kg}}\mathcal{I}_{k,g}(v_{k\cdot})^{1/\sigma_{kg}-1}\mathcal{I}_{k,M}(v_{k\cdot})^{-1}$, $s_{kg} = \sum_{j\in N_g} s_{kj} = \mathcal{I}_{k,g}(v_{k\cdot})^{1/\sigma_{kg}}\mathcal{I}_{k,M}(v_{k\cdot})^{-1}$, and $s_{kj|g} = \frac{s_{kj}}{s_{kg}} = e^{v_{kj}\sigma_{kg}}\mathcal{I}_{k,g}(v_{k\cdot})^{-1}$ where $s_{kj|g}$ denotes the share of workers of type k working in the firm j as a fraction of the total nest share.

The cross-wage super-elasticities in the Nested Logit model take the following form:

$$\zeta_{kjl} = \beta_{kj}\left[(1-\sigma_{kg})s_{kj|g}\frac{\mathcal{E}_{kjl|g}}{\mathcal{E}_{kj}} - s_{kj}\frac{\mathcal{E}_{kjl}}{\mathcal{E}_{kj}}\right] \tag{C.2}$$

where $\mathcal{E}_{kjl|g}$ denotes the within-nest cross-wage elasticities. The super-elasticity simplifies to:[5]

$$\zeta_{kj} = \beta_{kj}\left[\beta_{kj}(1-\sigma_{kg})s_{kj|g}(1-s_{kj|g})/\mathcal{E}_{kj} - s_{kj}\right]. \tag{C.3}$$

A direct application of Theorem 2 leads to the following result:

Corollary 1. *Whenever Assumptions 1, 2, and 3 (ii) hold and workers idiosyncratic utility shocks have a Nested Logit structure, an equilibrium exists and it is unique.*

The proof is immediate by showing that the sign restriction in Assumption 3 (i) holds in the Nested Logit Economy.

We can compare our framework to existing literature using this Nested Logit Economy. On one hand, Card et al. (2018) and Lamadon, Mogstad and Setzler (2022) consider a special case of imperfect competition which implies that the two indirect effects of changes in w_{kj} are null, i.e., $\frac{\partial \mathcal{I}_{k,g}(v_{k\cdot})}{\partial w_{kj}} f_2(.) + \frac{\partial \mathcal{I}_{k,M}(v_{k\cdot})}{\partial w_{kj}} f_3(.) = 0$. Such an assumption can considerably limit the effect of market power for some firms and impose important restrictions on the nature of strategic interactions. For instance, these frameworks assume away the possibility that some firms are dominant in a certain local market g, in such a way that they may hire a non-negligible share of some types of workers in their local market. Under this assumption, productivity or amenities shocks in firm j that affect w_{kj} do not have any spillover effects onto the equilibrium wage in a different firm j', $w_{kj'}$. Moreover, the atomistic firm assumption implies that $(1-\sigma_{kg})s_{kj|g} - s_{kj} = 0$ for all $(k,j) \in \mathcal{K}\times\mathcal{J}$, and $g \in \{1,...,G\}$. With $\sigma_{kg} > 1$, this implies that $s_{kj|g} = s_{kj} = 0$. Therefore, if we observe in the data that some firms have a significant share of type-k workers in their local market, i.e., $s_{kj|g} > \underline{s}$ for $\underline{s} > 0$, we can reject the atomistic firm assumption. Finally, we always have $[(1-\sigma_{kg})s_{kj|g} - s_{kj}] \leq 0$, which

[5]We could write also the elasticity as a function of the super-elasticity as in Edmond, Midrigan and Xu (2023), i.e., $\mathcal{E}_{kj} = \frac{\zeta_{kj}+\beta_{kj}s_{kj}}{\beta_{kj}^2(1-\sigma_{kg})s_{kj|g}(1-s_{kj|g})}$.

implies that the atomistic firm assumption leads to an overestimation of firms' labor supply elasticities—and thus the markdowns—and cross-wage super-elasticities.

On the other hand, Berger, Herkenhoff and Mongey (2022), impose the weaker condition that $\frac{\partial \mathcal{I}_{k,M}(v_{k\cdot})}{\partial w_{kj}} f_3(.) = 0$; in other words, they allow some firms to be dominant in their local market but no firm has enough power to hire a significant share of some type of workers at the aggregate market level.[6] Their restriction imposes that $s_{kj} = 0$ for all (k, j), but allows $(1 - \sigma_{kg})s_{kj|g} \neq 0$ for some (k, j). Therefore, they also tend to overestimate labor supply elasticities and cross-wage super-elasticities and thus the true markdowns but with a lower bias than the one estimated under the atomistic firm assumption.[7]

C.2. **Comparative statics: Passthrough.** To clarify how our comparative statics results generalize the special cases analyzed in the literature, we consider the Nested Logit Economy. In this case, the lower bound of Proposition 3(ii)-b simplifies to:

$$\left\{ \underbrace{\underbrace{1 - \frac{\beta_{kj}\sigma_{kg}}{\eta_{kj}}}_{LMS} - \beta_{kj}(1-\sigma_{kg})s_{kj|g}\left[\frac{1}{\eta_{kj}} + \beta_{kj}(1-s_{kj|g})\frac{(1-\mathrm{md}_{kj})^2}{\mathrm{md}_{kj}}\right]}_{BHM} + \beta_{kj}s_{kj}\left[\frac{1}{\eta_{kj}} + (1-\mathrm{md}_{kj})\right] \right\}^{-1} \tag{C.4}$$

LMS denotes the passthrough formula obtained in Lamadon, Mogstad and Setzler (2022) where firms are atomistic, i.e., $s_{kj|g} = s_{kj} \approx 0$. BHM represents the passthrough formula in the Berger, Herkenhoff and Mongey (2022) framework where strategic interactions channels are shut down, i.e., only one dominant firm per local market.[8] Here, our lower bound provides the general formula for the passthrough when all cross-wage elasticities and cross-wage super-elasticities are assumed to be zero, i.e., $\mathcal{E}_{kjl} = \zeta_{kjl} = 0$ for $l \neq j$, i.e., shutting down all strategic interaction channels. No specific restrictions are imposed on $\mathcal{E}_{kj}$ and ζ_{kj}.

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[6]In their context, this restriction arises as they consider a model with an infinite number of local markets.

[7]When firms compete according to Bertrand, the labor supply elasticity in Berger, Herkenhoff and Mongey (2022) is given by: $\mathcal{E}_{kj} = [\theta s_{kj|g} + \eta(1 - s_{kj|g})]$ which is a special case of our elasticity when $\theta = \beta_{kj}$, $\eta = \beta_{kj}\sigma_{kg}$ and $s_{kj} = 0$.

[8]In the Berger, Herkenhoff and Mongey (2022) case, the markdown is restricted to the case where $s_{kj} = 0$.

Appendix D. Additional derivations and results.

D.1. **Comparative Statics.** We exploit special features of the Jacobian of our model equilibrium to study comparative statics for the effect on equilibrium wages of changes in total factor productivity (TFP), amenities, and non-employment benefit shocks. We derive closed-form comparative statics for the duopsony version of our model and lower bounds for the general oligopsony version.

Recall the shorthand notation for the derivative of the log wage of type-k workers at firm j with respect to log wages of type-k workers at firm l:

$$\psi_{k,jl} = \frac{\partial \ln \mathrm{mpl}_{kj}}{\partial \ln w_{kl}} + \frac{\partial \ln \mathrm{md}_{kj}}{\partial \ln w_{kl}} \equiv \frac{\mathcal{E}_{kjl}}{\eta_{kj}} + (1 - \mathrm{md}_{kj})\zeta_{kjl}$$
$$\phi_{k,jl} = \frac{\partial \ln \mathrm{mpl}_{kj}}{\partial \ln u_{kl}} + \frac{\partial \ln \mathrm{md}_{kj}}{\partial \ln u_{kl}}.$$

Here is the complete version of Proposition 2 from the main text:

Proposition 3 (Comparative Statics)**.** *Consider that Assumptions 1, 2, and 3 hold. Let (s, w) denote the unique equilibrium outcome of our many-to-one matching model. In a neighborhood of the equilibrium (s, w) the following (general equilibrium) comparative statics hold:*

(i) **Duopsony***: $\mathcal{J} = \{j, l\}$. For any $k \in \mathcal{C}^j \cap \mathcal{C}^l$, we have*

(a)

$$\frac{w_{k0}}{w_{kj}} \frac{\partial w_{kj}}{\partial w_{k0}} = \frac{(1 - \psi_{k,ll})\psi_{k,j0} + \psi_{k,jl}\psi_{k,l0}}{(1 - \psi_{k,jj})(1 - \psi_{k,ll}) - \psi_{k,jl}\psi_{k,lj}} \geq 0.$$

(b) If the firms' production functions have a multiplicative structure of the form $F^l(.) = \check{\theta}_l \check{F}^l(.)$ where $\frac{\partial \check{F}^l(.)}{\partial \check{\theta}_l} = 0$, then for any $k \in \mathcal{C}^j \cap \mathcal{C}^l$, we have

$$\frac{\check{\theta}_l}{w_{kj}} \frac{\partial w_{kj}}{\partial \check{\theta}_l} = \frac{\psi_{k,jl}}{(1 - \psi_{k,jj})(1 - \psi_{k,ll}) - \psi_{k,jl}\psi_{k,lj}} \geq 0,$$
$$\frac{\check{\theta}_l}{w_{kl}} \frac{\partial w_{kl}}{\partial \check{\theta}_l} = \frac{(1 - \psi_{k,jj})}{(1 - \psi_{k,jj})(1 - \psi_{k,ll}) - \psi_{k,jl}\psi_{k,lj}} > 0.$$

(c)

$$\frac{u_{kl}}{w_{kj}} \frac{\partial w_{kj}}{\partial u_{kl}} = \frac{(1 - \psi_{k,ll})\phi_{k,jl} + \psi_{k,jl}\phi_{k,ll}}{(1 - \psi_{k,jj})(1 - \psi_{k,ll}) - \psi_{k,jl}\psi_{k,lj}} \gtreqless 0,$$
$$\frac{u_{kl}}{w_{kl}} \frac{\partial w_{kl}}{\partial u_{kl}} = \frac{(1 - \psi_{k,jj})\phi_{k,ll} + \psi_{k,lj}\phi_{k,jl}}{(1 - \psi_{k,jj})(1 - \psi_{k,ll}) - \psi_{k,jl}\psi_{k,lj}} \gtreqless 0.$$

(ii) **Oligopsony***: $J \geq 2$. For any $k \in \mathcal{C}^j \cap \mathcal{C}^l$, and $l, j \in \mathcal{J}$, we have*

(a) For any $k \in \mathcal{C}^j$, we have:

$$\frac{w_{k0}}{w_{kj}}\frac{\partial w_{kj}}{\partial w_{k0}} \geq \frac{\mathcal{E}_{kj0}/\eta_{kj} + (1 - md_{kj})\zeta_{kj0}}{1 - \mathcal{E}_{kj}/\eta_{kj} - (1 - md_{kj})\zeta_{kj}} \geq 0.$$

(b) If the firms' production functions have a multiplicative structure of the form $F^l(.) = \check{\theta}_l \check{F}^l(.)$ where $\frac{\partial \check{F}^l(.)}{\partial \check{\theta}_l} = 0$, then for any $k \in \mathcal{C}^j \cap \mathcal{C}^l$, we have:

$$\frac{\check{\theta}_l}{w_{kj}}\frac{\partial w_{kj}}{\partial \check{\theta}_l} \begin{cases} \geq \frac{\mathcal{E}_{kjl}/\eta_{kj} + (1-md_{kj})\zeta_{kjl}}{\left(1-\mathcal{E}_{kj}/\eta_{kj} - (1-md_{kj})\zeta_{kj}\right)(1-\mathcal{E}_{kl}/\eta_{kl} - (1-md_{kl})\zeta_{kl})} \geq 0 \text{ if } j \neq l, \\ \geq \frac{1}{(1-\mathcal{E}_{kl}/\eta_{kl} - (1-md_{kl})\zeta_{kl})} > 0, \text{ if } j = l. \end{cases}$$

where $\psi_{k,jl}, \phi_{k,jl} \geq 0$ for $l \neq j$, and $\psi_{k,ll}, \phi_{k,ll} \leq 0$.

Before detailing the proof of Proposition 3, we discuss the intuition behind the comparative statics for non-employment benefit and amenities shocks.

Non-employment benefit shocks. Proposition 3(i)/(ii)-a shows the effect of an exogenous increase of non-employment benefits on the equilibrium wages. The equation in (i)-a shows explicitly the different channels by which an exogenous shock to non-employment benefits affects the equilibrium wages in the duopsony case: An increase of w_{k0} has a direct effect on mpl_{kj} and md_{kj}, and firm j increases w_{kj} in response. An indirect effect is transmitted through firm l: The increase of w_{k0} has a direct effect also on mpl_{kl} and md_{kl}, and firm l increases w_{kl}. This change in w_{kl} affects firm j through $\psi_{k,jl}$ and firm j responds by increasing w_{kj}. This in turn generates a response of firm l through $\psi_{k,lj}$, and so on. This succession of responses converges and leads to a final total increase of equilibrium wages. In sum, the strategic responses are mediated by $\psi_{k,jl}$ and $\psi_{k,lj}$ in the duopsony context.

In the more general case with $J \geq 2$, the strategic interactions are captured by $\psi_{k,jr}$ and $\psi_{k,rj}$ for all $r \in \mathcal{J} \setminus \{j\}$. Proposition 3(ii)-a shows that the indirect effects due to strategic interactions can only amplify the magnitudes of the effect of an exogenous increase of non-employment benefits on the equilibrium wages. Indeed, the lower bound derived in (ii)-a is achieved when there are no strategic interactions, i.e., $\psi_{k,jr} = \psi_{k,rj} = 0$ for all $r \in \mathcal{J} \setminus \{j\}$, which happens for example under the "atomistic" firms assumption imposed in Card et al. (2018) and Lamadon, Mogstad and Setzler (2022) or in the Berger, Herkenhoff and Mongey (2022) framework where each local market contains only one firm.

Amenities shocks. In Proposition 3(i)-c we show the effect of a positive increase of type-k worker preference for firm l amenities on equilibrium wages. The duopsony case shows that in the case of an amenities shock, the indirect effect due to strategic interactions works against the direct effect and does not allow us to determine the sign of the equilibrium effect. An increase of u_{kl} directly affects mpl_{kl} and md_{kl} through $\phi_{k,ll}$ and causes firm l to lower the wage w_{kl}. At the same time, the increase in u_{kl} directly affects mpl_{kj} and md_{kj} through $\phi_{k,jl}$, leading firm j to increase the wage w_{kj} though a competition effect. The opposite changes in w_{kl} and w_{kj} both firms through $\psi_{k,jl}$ and $\psi_{k,lj}$. After a set of iterative responses we have the final effect on equilibrium wages and the net sign of this effect is ambiguous. When the strategic interaction terms are 0, i.e., $\psi_{k,jl} = \psi_{k,lj} = 0$, we have $\frac{u_{kl}}{w_{kl}} \frac{\partial w_{kl}}{\partial u_{kl}} < 0$. But when $\psi_{k,jl}$ and $\psi_{k,lj}$ are not null, the resulting aggregate effect could be positive.

D.1.1. *Proof of Proposition 3.* Under Assumptions 1, 2, and 3, we proved that we have an unique equilibrium w^{eq} such that $w^{eq} = B(w^{eq})$. For sake of simplicity let us ignore the upper-script eq in the rest of the proof. By the Implicit Function Theorem we have:

$$\frac{dw}{dw_{k0}} = \mathbb{J}_\delta^{-1}(w) \frac{\partial B(w)}{\partial w_{k0}},$$
$$\frac{dw}{d\gamma_{kl}} = \mathbb{J}_\delta^{-1}(w) \frac{\partial B(w)}{\partial \gamma_{kl}},$$
$$\frac{dw}{d\theta_l} = \mathbb{J}_\delta^{-1}(w) \frac{\partial B(w)}{\partial \theta_l}.$$

Under Assumption 3, $\mathbb{J}_\delta(w)$ is a block diagonal matrix, more precisely it can be written

$$\underset{(KJ\times KJ)}{\mathbb{J}_\delta(w)} = \begin{pmatrix} \mathbb{J}_{\delta,1\cdot}(w) & 0 & \cdots & 0 \\ 0 & \mathbb{J}_{\delta,2\cdot}(w) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbb{J}_{\delta,K\cdot}(w) \end{pmatrix} \text{ where } \underset{(J\times J)}{\mathbb{J}_{\delta,k\cdot}(w)} = \begin{pmatrix} \frac{\partial \delta_{k1}}{\partial w_{k1}} & \cdots & \frac{\partial \delta_{k1}}{\partial w_{kJ}} \\ \vdots & \ddots & \vdots \\ \frac{\partial \delta_{kJ}}{\partial w_{k1}} & \cdots & \frac{\partial \delta_{kJ}}{\partial w_{kJ}} \end{pmatrix}.$$

Case 1 of the proof of Theorem 2 shows that each $\mathbb{J}_{\delta,k\cdot}(w)$ for $k \in \mathcal{K}$ is positive diagonally dominant, therefore its inverse exists and then we have

$$\underset{(KJ\times KJ)}{\mathbb{J}_\delta^{-1}(w)} = \begin{pmatrix} \mathbb{J}_{\delta,1\cdot}^{-1}(w) & 0 & \cdots & 0 \\ 0 & \mathbb{J}_{\delta,2\cdot}^{-1}(w) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbb{J}_{\delta,K\cdot}^{-1}(w) \end{pmatrix}.$$

We then have $\frac{dw_{m\cdot}}{dw_{k0}} = \mathbb{J}_{\delta,m\cdot}^{-1}(w) \frac{\partial B_{m\cdot}(w)}{\partial w_{k0}}$ where $w_{m\cdot} = \begin{pmatrix} w_{m1} \\ \vdots \\ w_{mJ} \end{pmatrix}$, and $B_{m\cdot}(w) = \begin{pmatrix} B_{m1}(w) \\ \vdots \\ B_{mJ}(w) \end{pmatrix}$.

Our derived bounds come from the linear algebra results on M-matrices and inverse M-matrices, i.e., Carlson and Markham (1979); Fiedler and Pták (1962). In fact, case 1 of the Proof of Theorem 2, shows that any $\mathbb{J}_{\delta,k\cdot}(w)$ for $k \in \mathcal{K}$ is positive diagonally dominant and have non-positive off diagonal elements. Then, $\mathbb{J}_{\delta,k\cdot}(w)$, and $\mathbb{J}_\delta(w)$ are M Matrices. Our proofs widely use the result (4.2) of Fiedler and Pták (1962), which states that if A and B are two M matrices such that $A \leqq B$, then $A^{-1} \geqq B^{-1} \geqq 0$. Let's denote by DA the diagonal matrix formed by the diagonal elements of the matrix A. Under Assumption 3, we have $\mathbb{J}_{\delta,k\cdot}(w) \leq D\mathbb{J}_{\delta,k\cdot}(w) \Rightarrow \mathbb{J}^{-1}_{\delta,k\cdot}(w) \geq [D\mathbb{J}_{\delta,k\cdot}(w)]^{-1} \Rightarrow \mathbb{J}^{-1}_{\delta,k\cdot}(w)\frac{\partial B_{k\cdot}(w)}{\partial w_{k0}} \geq [D\mathbb{J}_{\delta,k\cdot}(w)]^{-1}\frac{\partial B_{k\cdot}(w)}{\partial w_{k0}}$ where the last inequality holds since $\frac{\partial B_{kj}(w)}{\partial w_{k0}} \geq 0$ under Assumption 3.

It follows from the latter inequality that:

$$\frac{\partial w_{kj}}{\partial w_{k0}} \geq \frac{w_{kj}}{w_{k0}}\frac{\psi_{k,j0}}{1-\psi_{k,jj}} \geq 0$$

where $\psi_{k,jl} = \left(\frac{w_{kl}}{\ell_{kj}}\frac{\partial \ell_{kj}(w_{k\cdot})}{\partial w_{kl}}\left(\frac{F^j_{kk}}{F^j_k}\ell_{kj}\right) + \frac{1}{(1+\mathcal{E}_{kj}(w_{k\cdot}))}\frac{w_{kl}}{\mathcal{E}_{kj}(w_{k\cdot})}\frac{\partial \mathcal{E}_{kj}(w_{k\cdot})}{\partial w_{kl}}\right)$.
This latter inequality becomes evident as soon as you remark that:

$$\frac{\partial \delta_{kj}}{\partial w_{kl}} \begin{cases} -\left(\frac{w_{kj}}{w_{kl}}\right)\psi_{k,jl} \text{ if } j \neq l \\ 1-\psi_{k,jl} \text{ if } j = l \end{cases}$$

This proves the first set of bounds.

Second, for $a_{ll} > 0$ and $a_{jl} \leq 0$ when $j \neq l$ it can be shown that

$$\begin{aligned}
H^{-1}(a_{\cdot\cdot}) &\equiv \begin{pmatrix}
a_{11} & 0 & \cdots & 0 & a_{1l} & 0 & \cdots & 0 \\
\vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\
0 & \cdots & \cdots & 0 & a_{ll} & 0 & \cdots & 0 \\
0 & \cdots & \cdots & \cdots & 0 & a_{l+1,l+1} & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
0 & \cdots & \cdots & \cdots & \cdots & \cdots & 0 & a_{J,J}
\end{pmatrix}^{-1} \\
&= \begin{pmatrix}
1/a_{11} & 0 & \cdots & 0 & -a_{1l}/a_{11}a_{ll} & 0 & \cdots & 0 \\
\vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\
0 & \cdots & \cdots & 0 & 1/a_{ll} & 0 & \cdots & 0 \\
0 & \cdots & \cdots & \cdots & 0 & 1/a_{l+1,l+1} & \cdots & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
0 & \cdots & \cdots & \cdots & \cdots & \cdots & 0 & 1/a_{J,J}
\end{pmatrix}, \\
\frac{\partial B_{k\cdot}(w)}{\partial \theta_l} &= \begin{pmatrix} 0 \\ \vdots \\ 0 \\ B_{kl}(w)/\theta_l \\ 0 \\ \vdots \\ 0 \end{pmatrix} \geq 0.
\end{aligned}$$

For $a_{jl} \equiv \frac{\partial \delta_{kj}}{\partial w_{kl}}$, we have

$$\mathbb{J}_{\delta,k\cdot}(w) \leq H\left(\frac{\partial \delta_{k\cdot}}{\partial w_{k\cdot}}\right) \Rightarrow \mathbb{J}^{-1}_{\delta,k\cdot}(w) \geq \left[H\left(\frac{\partial \delta_{k\cdot}}{\partial w_{k\cdot}}\right)\right]^{-1} \Rightarrow \mathbb{J}^{-1}_{\delta,k\cdot}(w)\frac{\partial B_{k\cdot}(w)}{\partial \theta_l} \geq \left[H\left(\frac{\partial \delta_{k\cdot}}{\partial w_{k\cdot}}\right)\right]^{-1}\frac{\partial B_{k\cdot}(w)}{\partial \theta_l}.$$

The latter inequality implies that for $j \leq l$ we have:

$$\frac{\partial w_{kj}}{\partial \theta_l}\begin{cases} \geq -\frac{\frac{\partial \delta_{kj}}{\partial w_{kl}}}{\frac{\partial \delta_{kj}}{\partial w_{kj}}\frac{\partial \delta_{kl}}{\partial w_{kl}}}\frac{B_{kl}(w)}{\theta_l} = \frac{w_{kj}\psi_{k,jl}}{\theta_l(1-\psi_{k,jj})(1-\psi_{k,ll})} \geq 0 \text{ if } j<l \\ \geq \frac{1}{\frac{\partial \delta_{kl}}{\partial w_{kl}}}\frac{B_{kl}(w)}{\theta_l} = \frac{w_{kl}}{\theta_l(1-\psi_{k,ll})} > 0, \text{ if } j=l. \text{ otherwise.}\end{cases} \tag{D.1}$$

For $j<l$, we can follow the same process by considering H as a lower triangular matrix. The exact same proof holds for $\frac{\partial w_{kj}}{\partial \theta_l}$. This completes the proof.

Special case: Duopsony. In this special case, we could have a passthrough formula that will hold at equality. This will allow us to have an intuition of the shock transmission from a firm j to a firm l. Recall that $\frac{dw_{m\cdot}}{dw_{k0}} = \mathbb{J}^{-1}_{\delta,m\cdot}(w)\frac{\partial B_{m\cdot}(w)}{\partial w_{k0}}$, and $\frac{\partial \delta_{kj}}{\partial w_{kl}} = -\left(\frac{w_{kj}}{w_{kl}}\right)\psi_{k,jl}$ for $l \neq j$.

Now, consider that $\mathcal{J} = \{j,l\}$. In this special case the inverse of the Jacobian matrix is given by:

$$(\mathbb{J}_{\delta,k\cdot}(w))^{-1} = \begin{pmatrix} \frac{\partial \delta_{kj}}{\partial w_{kj}} & \frac{\partial \delta_{kj}}{\partial w_{kl}} \\ \frac{\partial \delta_{kl}}{\partial w_{kj}} & \frac{\partial \delta_{kl}}{\partial w_{kl}}\end{pmatrix}^{-1} = \frac{1}{(1-\psi_{k,jj})(1-\psi_{k,ll})-\psi_{k,jl}\psi_{k,lj}}\begin{pmatrix}(1-\psi_{k,ll}) & \left(\frac{w_{kj}}{w_{kl}}\right)\psi_{k,jl} \\ \left(\frac{w_{kl}}{w_{kj}}\right)\psi_{k,lj} & (1-\psi_{k,jj})\end{pmatrix}.$$

Then, we can easily derive the following:

$$\frac{w_{k0}}{w_{kj}}\frac{\partial w_{kj}}{\partial w_{k0}} = \frac{(1-\psi_{k,ll})\psi_{k,j0}+\psi_{k,jl}\psi_{k,l0}}{(1-\psi_{k,jj})(1-\psi_{k,ll})-\psi_{k,jl}\psi_{k,lj}} \geq 0 \tag{D.2}$$

$$\frac{u_{kl}}{w_{kj}}\frac{\partial w_{kj}}{\partial u_{kl}} = \frac{(1-\psi_{k,ll})\phi_{k,jl}+\psi_{k,jl}\phi_{k,ll}}{(1-\psi_{k,jj})(1-\psi_{k,ll})-\psi_{k,jl}\psi_{k,lj}} \gtreqless 0 \tag{D.3}$$

$$\frac{u_{kl}}{w_{kl}}\frac{\partial w_{kl}}{\partial u_{kl}} = \frac{(1-\psi_{k,jj})\phi_{k,ll}+\psi_{k,lj}\phi_{k,jl}}{(1-\psi_{k,jj})(1-\psi_{k,ll})-\psi_{k,jl}\psi_{k,lj}} \gtreqless 0 \tag{D.4}$$

$$\frac{\theta_l}{w_{kj}}\frac{\partial w_{kj}}{\partial \theta_l} = \frac{\psi_{k,jl}}{(1-\psi_{k,jj})(1-\psi_{k,ll})-\psi_{k,jl}\psi_{k,lj}} \geq 0 \tag{D.5}$$

$$\frac{\theta_l}{w_{kj}}\frac{\partial w_{kj}}{\partial \theta_l} = \frac{(1-\psi_{k,jj})}{(1-\psi_{k,jj})(1-\psi_{k,ll})-\psi_{k,jl}\psi_{k,lj}} \geq 0 \tag{D.6}$$

where the signs restrictions hold, because $\psi_{k,jl}, \phi_{k,jl} \geq 0$ for $l \neq j$, and $\psi_{k,ll}, \phi_{k,ll} \leq 0$ with $\phi_{k,jl} = \left(\frac{u_{kl}}{\ell_{kj}}\frac{\partial \ell_{kj}(w_{k\cdot})}{\partial u_{kl}}\left(\frac{F^j_{kk}}{F^j_k}\ell_{kj}\right) + \frac{1}{(1+\mathcal{E}_{kj}(w_{k\cdot}))}\frac{u_{kl}}{\mathcal{E}_{kj}(w_{k\cdot})}\frac{\partial \mathcal{E}_{kj}(w_{k\cdot})}{\partial u_{kl}}\right)$.

D.2. **Recovering unobserved types.** The proposed identification strategy requires us to observe at least two time periods. We consider the following potential outcomes model:

$$Y_{it} = \sum_{j \in \mathcal{J}_0}[\ln w_{\mathbf{k}jt} + \eta_{ijt}]1\{D_{it}=j\}, \quad t \in \{1,...,T\} \tag{D.7}$$

where Y_{it} denotes the observed log earnings of individual i at time t, and $1\{\cdot\}$ denotes the indicator function. $Y_{ijt} \equiv \ln w_{\mathbf{k}jt} + \eta_{ijt}$ denotes potential log earnings if individual i was externally assigned to work at firm j in period t. The potential outcomes are decomposed into two parts (i) $\ln w_{\mathbf{k}jt}$ is the log equilibrium wage, and (ii) η_{ijt} is measurement error or an i.i.d. worker-firm match effect realized after potential mobility across periods.

While in the main text we assumed that the worker's type k is observed by both firms and the econometrician, in general, we could allow k to consist of two subgroups of types, i.e., $k \equiv (\bar{k}, \tilde{k})$, where $\bar{k}$ is defined based on the underlying vector of characteristics $\overline{X}$ that are observed both by the econometrician and firms while $\tilde{k}$ is defined based on the set of characteristics $\widetilde{X}$ that are observable only to firms but not to the econometrician.

Let m_{it} denote the mobility variable, more precisely $m_{it} = 1$ iff $D_{it} \neq D_{it+1}$, i.e., $m_{it} = 1\{D_{it} \neq D_{it+1}\}$. Using shorthand notation $\bar{\mathbf{k}}^{\mathbf{t+1}} = (\bar{\mathbf{k}}_{\mathbf{t}}, \bar{\mathbf{k}}_{\mathbf{t+1}})$, consider the following assumption:

Assumption 4 (Time invariance, Mobility, and Serial Dependence)**.** *We impose the following restrictions.*

(i) Time invariance of unobserved types: $\tilde{\mathbf{k}}_{\mathbf{t}} = \tilde{\mathbf{k}}$ *for* $t \in \{1, ..., T\}$.

(ii) Classical errors: $(\eta_{ijt}, \eta_{ilt+1}) \perp (D_{it}, D_{it+1}) | \tilde{\mathbf{k}}, \bar{\mathbf{k}}_{\mathbf{t}}, \bar{\mathbf{k}}_{\mathbf{t+1}}$

(iii) No serial dependence in the errors: $\eta_{ijt} \perp \eta_{ilt+1} | \tilde{\mathbf{k}}, \bar{\mathbf{k}}_{\mathbf{t}}, \bar{\mathbf{k}}_{\mathbf{t+1}}$ *and* $\eta_{ijt} \perp \bar{\mathbf{k}}_{\mathbf{t+1}} | \tilde{\mathbf{k}}, \bar{\mathbf{k}}_{\mathbf{t}}$

Assumption 4(i) requires the unobserved types to be time invariant. In the same spirit as Burdett and Mortensen (1998) and Hagedorn, Law and Manovskii (2017), Assumption 4(ii) requires the errors to not be correlated with sorting and mobility decisions. The intuition is that these errors are realized after the matches between workers and firms have been formed. Assumption 4(iii) requires the measurement errors associated to a specific mover to not be serially dependent.

Under Assumption 4 we can show that

$$
\begin{aligned}
&\mathbb{P}(Y_{it} \leq y_t, Y_{i,t+1} \leq y_{t+1} | D_{it} = j, D_{it+1} = l, m_{it} = 1, \bar{\mathbf{k}}^{\mathbf{t+1}} = \bar{k}^{t+1}) \\
&\qquad = \sum_{\tilde{k}} \mathbb{P}_{\tilde{k}j}(y_t | \bar{k}_t) \mathbb{P}^m_{\tilde{k}l}(y_{t+1} | \bar{k}^{t+1}) \mathbb{P}(\tilde{\mathbf{k}} = \tilde{k} | D_{it} = j, D_{it+1} = l, m_{it} = 1, \bar{\mathbf{k}}^{\mathbf{t+1}} = \bar{k}^{t+1}) \quad \text{(D.8)}
\end{aligned}
$$

where

$$
\begin{aligned}
\mathbb{P}_{\tilde{k}j}(y_t | \bar{k}_t) &\equiv \mathbb{P}(Y_{it} \leq y_t | D_{it} = j, \tilde{\mathbf{k}} = \tilde{k}, \bar{\mathbf{k}}_{\mathbf{t}} = \bar{k}_t), && \text{(D.9)} \\
\mathbb{P}^m_{\tilde{k}l}(y_{t+1} | \bar{k}^{t+1}) &\equiv \mathbb{P}(Y_{i,t+1} \leq y_{t+1} | D_{it+1} = l, m_{it} = 1, \tilde{\mathbf{k}} = \tilde{k}, \bar{\mathbf{k}}^{\mathbf{t+1}} = \bar{k}^{t+1}). && \text{(D.10)}
\end{aligned}
$$

Whenever the above decomposition holds and the following three requirements hold: (i) Any two firms j and l belong to a connecting cycle as formally defined in Bonhomme, Lamadon and Manresa (2019), Definition 1, (ii) there exists some asymmetry in the worker type composition between different firms, i.e, Bonhomme, Lamadon and Manresa (2019), Assumption 3(i), and (iii) the matrix defined by the joint log earning distribution $\mathbb{P}(Y_{it} \leq y_t, Y_{i,t+1} \leq y_{t+1} | D_{it} = j, D_{it+1} = l, m_{it} = 1, \bar{\mathbf{k}}^{\mathbf{t+1}} = \bar{k}^{t+1})$ for different values of (y_t, y_{t+1}) respects a certain rank condition, i.e, Bonhomme, Lamadon and Manresa (2019), Assumption 3(ii). Then Theorem 1 of Bonhomme, Lamadon and Manresa (2019) applies and the following quantities are point identified: $\mathbb{P}_{\tilde{k}j}(y_t|\bar{k}_t)$, $\mathbb{P}^m_{\tilde{k}l}(y_{t+1}|\bar{k}_{t+1})$, and $\mathbb{P}_{jt}(\tilde{k}|\bar{k}_t) \equiv \mathbb{P}(\tilde{\mathbf{k}} = \tilde{k}|D_{it} = j, \bar{\mathbf{k}}_{\mathbf{t}} = \bar{k}_t)$.

These distributions can be parametrically estimated using the EM algorithm entertained in Bonhomme, Lamadon and Manresa (2019). Using this identification result, it is possible to recover equilibrium wages and shares that were initially unobserved to the econometrician. More precisely, we have the following result:

Proposition 4 (Identification of equilibrium wages and shares)**.** *Consider Assumption 4 holds, and the cdf of classical errors $F_{\eta_{ijt}|\mathbf{k_t}=k_t}(.)$, and $F_{\eta_{ilt+1}|\mathbf{k^{t+1}}=k^{t+1}}(.)$ are known and strictly increasing on $\mathbb{R}$. If the following quantities are point identified $\mathbb{P}_{\tilde{k}j}(y_t|\bar{k}_t)$, $\mathbb{P}^m_{\tilde{k}l}(y_{t+1}|\bar{k}_{t+1})$, $\mathbb{P}_{jt}(\tilde{k}|\bar{k}_t)$; then we have the following identification result:*

$$w_{kjt} = \exp\left\{y_t - F^{-1}_{\eta_{ijt}|\mathbf{k_t}=k_t}\left(\mathbb{P}_{\tilde{k}j}(y_t|\bar{k}_t)\right)\right\}, \tag{D.11}$$

$$w_{klt+1} = \exp\left\{y_{t+1} - F^{-1}_{\eta_{ilt+1}|\mathbf{k^{t+1}}=k^{t+1}}\left(\mathbb{P}^m_{\tilde{k}l}(y_{t+1}|\bar{k}_{t+1})\right)\right\}, \tag{D.12}$$

$$s_{kjt} = \mathbb{P}_{jt}(\tilde{k}|\bar{k}_t)\frac{s_{\bar{k}jt}}{\sum_{\mathcal{J}_0}\mathbb{P}_{jt}(\tilde{k}|\bar{k}_t)s_{\bar{k}jt}}. \tag{D.13}$$

where $s_{kjt} = \mathbb{P}(D_{it} = j|\mathbf{k_t} = k_t)$ and $s_{\bar{k}jt} = \mathbb{P}(D_{it} = j|\bar{\mathbf{k}}_{\mathbf{t}} = \bar{k}_t)$.

Proof of Proposition 4.

$$\begin{aligned}
&\mathbb{P}(Y_{it} \leq y_t, Y_{i,t+1} \leq y_{t+1}|D_{it} = j, D_{it+1} = l, m_{it} = 1, \bar{\mathbf{k}}^{\mathbf{t+1}} = \bar{k}^{t+1}) = \\
&\quad = \sum_{\tilde{k}} \mathbb{P}(Y_{it} \leq y_t, Y_{i,t+1} \leq y_{t+1}|D_{it} = j, D_{it+1} = l, \tilde{\mathbf{k}} = \tilde{k}, \bar{\mathbf{k}}^{\mathbf{t+1}} = \bar{k}^{t+1}) \\
&\qquad \times \underbrace{\mathbb{P}(\tilde{\mathbf{k}} = \tilde{k}|D_{it} = j, D_{it+1} = l, \bar{\mathbf{k}}_{\mathbf{t}} = \bar{k}_t, \bar{\mathbf{k}}_{\mathbf{t+1}} = \bar{k}_{t+1})}_{P(\tilde{k}|j,l,\bar{k}^{t+1})}
\end{aligned}$$

$$\begin{aligned}
&= \sum_{\tilde{k}} \mathbb{P}(\ln w_{\mathbf{k}jt} + \eta_{ijt} \leq y_t, \ln w_{\mathbf{k}j,t+1} + \eta_{ilt+1} \leq y_{t+1}|D_{it} = j, D_{it+1} = l, \tilde{\mathbf{k}} = \tilde{k}, \bar{\mathbf{k}}^{\mathbf{t+1}} = \bar{k}^{t+1}) \times P(\tilde{k}|j,l,\bar{k}^{t+1}) \\
&= \sum_{\tilde{k}} \mathbb{P}\left(\ln w_{\mathbf{k}jt} + \eta_{ijt} \leq y_t, \ln w_{\mathbf{k}j,t+1} + \eta_{ilt+1} \leq y_{t+1}|\tilde{\mathbf{k}} = \tilde{k}, \bar{\mathbf{k}}^{\mathbf{t+1}} = \bar{k}^{t+1}\right) \times P(\tilde{k}|j,l,\bar{k}^{t+1}) \\
&= \sum_{\tilde{k}} \mathbb{P}\left(\ln w_{\mathbf{k}jt} + \eta_{ijt} \leq y_t|\tilde{\mathbf{k}} = \tilde{k}, \bar{\mathbf{k}}^{\mathbf{t+1}} = \bar{k}^{t+1}\right) \times \mathbb{P}\left(\ln w_{\mathbf{k}j,t+1} + \eta_{ilt+1} \leq y_{t+1}|\tilde{\mathbf{k}} = \tilde{k}, \bar{\mathbf{k}}^{\mathbf{t+1}} = \bar{k}^{t+1}\right) \times P(\tilde{k}|j,l,\bar{k}^{t+1}) \\
&= \sum_{\tilde{k}} \mathbb{P}\left(\ln w_{\mathbf{k}jt} + \eta_{ijt} \leq y_t, \ln w_{\mathbf{k}j,t+1} + \eta_{ilt+1} \leq y_{t+1}|\tilde{\mathbf{k}} = \tilde{k}, \bar{\mathbf{k}}^{\mathbf{t+1}} = \bar{k}^{t+1}\right) \times P(\tilde{k}|j,l,\bar{k}^{t+1}) \\
&= \sum_{\tilde{k}} \mathbb{P}\left(Y_{it} \leq y_t|D_{it} = j, \tilde{\mathbf{k}} = \tilde{k}, \bar{\mathbf{k}}_{\mathbf{t}} = \bar{k}_t\right) \times \mathbb{P}\left(Y_{i,t+1} \leq y_{t+1}|D_{it+1} = l, m_{it} = 1, \tilde{\mathbf{k}} = \tilde{k}, \bar{\mathbf{k}}^{\mathbf{t+1}} = \bar{k}^{t+1}\right) \times P(\tilde{k}|j,l,\bar{k}^{t+1})
\end{aligned}$$

Now, we have

$$\begin{aligned}\mathbb{P}_{\tilde{k}j}(y_t|\bar{k}_t) &\equiv \mathbb{P}(Y_{it} \leq y_t|D_{it}=j, \tilde{\mathbf{k}}=\tilde{k}, \bar{\mathbf{k}}_{\mathbf{t}}=\bar{k}_t)\\ &= \mathbb{P}(\ln w_{\mathbf{k}jt}+\eta_{ijt} \leq y_t|D_{it}=j, \tilde{\mathbf{k}}=\tilde{k}, \bar{\mathbf{k}}_{\mathbf{t}}=\bar{k}_t)\\ &= \mathbb{P}(\ln w_{\mathbf{k}jt}+\eta_{ijt} \leq y_t|\tilde{\mathbf{k}}=\tilde{k}, \bar{\mathbf{k}}_{\mathbf{t}}=\bar{k}_t)\\ &= \mathbb{P}(\eta_{ijt} \leq y_t-\ln w_{\mathbf{k}jt}|\tilde{\mathbf{k}}=\tilde{k}, \bar{\mathbf{k}}_{\mathbf{t}}=\bar{k}_t) = F_{\eta_{ijt}|\bar{\mathbf{k}}_{\mathbf{t}}=\bar{k}_t}(y_t-\ln w_{\mathbf{k}jt})\end{aligned}$$

We can then easily recover the first result by inverting the last equation and obtain:

$$w_{kjt} = \exp\left\{y_t - F^{-1}_{\eta_{ijt}|\bar{\mathbf{k}}_{\mathbf{t}}=\bar{k}_t}\left(\mathbb{P}_{\tilde{k}j}(y_t|\bar{k}_t)\right)\right\}.$$

The second equality of the proposition could be derived analogously. For the last equality we have:

$$\begin{aligned}\mathbb{P}(D_{it}=j|\tilde{\mathbf{k}}=\tilde{k}, \bar{\mathbf{k}}_{\mathbf{t}}=\bar{k}_t) &= \frac{\mathbb{P}(\tilde{\mathbf{k}}=\tilde{k}|D_{it}=j, \bar{\mathbf{k}}_{\mathbf{t}}=\bar{k}_t)\times\mathbb{P}(D_{it}=j|\tilde{\mathbf{k}}=\tilde{k}, \bar{\mathbf{k}}_{\mathbf{t}}=\bar{k}_t)}{\mathbb{P}(\tilde{\mathbf{k}}=\tilde{k}|\bar{\mathbf{k}}_{\mathbf{t}}=\bar{k}_t)}\\ &= \frac{\mathbb{P}(\tilde{\mathbf{k}}=\tilde{k}|D_{it}=j, \bar{\mathbf{k}}_{\mathbf{t}}=\bar{k}_t)\times\mathbb{P}(D_{it}=j|\tilde{\mathbf{k}}=\tilde{k}, \bar{\mathbf{k}}_{\mathbf{t}}=\bar{k}_t)}{\sum_j \mathbb{P}(\tilde{\mathbf{k}}=\tilde{k}|D_{it}=j, \bar{\mathbf{k}}_{\mathbf{t}}=\bar{k}_t)\times\mathbb{P}(D_{it}=j|\tilde{\mathbf{k}}=\tilde{k}, \bar{\mathbf{k}}_{\mathbf{t}}=\bar{k}_t)}\end{aligned}$$

□

Parametric estimation and EM algorithm. For practical purposes, we impose a normality distribution for the classical errors, then $\ln w_{kjt}+\eta_{ijt}|\mathbf{k^t}=k^t \sim N(\ln w_{kjt}, \varrho_{kjt})$ and $\ln w_{klt}+\eta_{ilt+1}|\mathbf{k^{t+1}}=k^{t+1} \sim N(\ln w_{klt+1}, \varrho_{klt+1})$. Let $\tilde{K}$ denote the number of unobserved types, $C_{\bar{k}^t}$ be a set of firms that have been hiring workers of observable types $\bar{k}^t$ over the two periods t and $t+1$ and belonging to a connecting cycle as defined in Bonhomme, Lamadon and Manresa (2019). $N^m_{\bar{k}^t}$ denotes the number of movers with observable types $\bar{k}^t$. First, we consider the following log-likelihood function for job movers:

$$\sum_{i=1}^{N^m_{\bar{k}^t}}\sum_{j\in C_{\bar{k}^t}}\sum_{l\in C_{\bar{k}^t}}\ln\left(\sum_{\tilde{k}=1}^{\tilde{K}} p_{\tilde{k}jl}\frac{1}{\sqrt{4\pi^2\varrho_{(\tilde{k},\bar{k}_t)jt}\varrho_{(\tilde{k},\bar{k}_t)lt+1}}}e^{-\frac{\left(y_{it}-\ln w_{(\tilde{k},\bar{k}_t)jt}\right)^2}{2\varrho^2_{(\tilde{k},\bar{k}_t)jt}}-\frac{\left(y_{it+1}-\ln w_{(\tilde{k},\bar{k}_t)lt+1}\right)^2}{2\varrho^2_{(\tilde{k},\bar{k}_t)lt+1}}}\right) \tag{D.14}$$

where $\hat{w}_{(\tilde{k},\bar{k}_t)jt}$, $\hat{w}_{(\tilde{k},\bar{k}_t)lt+1}$, $\hat{\varrho}_{(\tilde{k},\bar{k}_t)jt}$, $\hat{\varrho}_{(\tilde{k},\bar{k}_t)lt+1}$, and $\hat{p}_{\tilde{k}jl}$ for $\tilde{k}=1,...,\tilde{K}$ are estimated by maximizing (D.15) using the EM algorithm.

Second, we consider the log-likelihood of the for all workers at the period t:

$$\sum_{i=1}^{N_{\bar{k}^t}}\sum_{j\in C_{\bar{k}_t}}\ln\left(\sum_{\tilde{k}=1}^{\tilde{K}} q_{\tilde{k}jt}\frac{1}{\sqrt{4\pi^2\hat{\varrho}_{(\tilde{k},\bar{k}_t)jt}}}e^{-\frac{\left(y_{it}-\ln\hat{w}_{(\tilde{k},\bar{k}_t)jt}\right)^2}{2\hat{\varrho}^2_{(\tilde{k},\bar{k}_t)jt}}}\right) \tag{D.15}$$

where $N_{\bar{k}_t}$ denotes the number of workers with observable types $\bar{k}_t$, and $q_{\tilde{k}jt} \equiv \mathbb{P}_{jt}(\tilde{k}|\bar{k}_t)$. Again we estimate $\hat{q}_{\tilde{k}jt}$ by maximizing equation (D.15) using the EM algorithm. Then we use equation (D.13) to recover $\hat{s}_{kjt}$.

Given employment shares s_{kjt} for each firm and worker type, we can then obtain the total quantity of each worker type in the population, $m_{kt} = \sum_j \ell_{kjt}$, as the (year-by-year) solution to an overdetermined system of linear equations: $S_t m_t = \mu_t$. Here S_t is the known $J \times K$ matrix of worker type shares s_{kjt} at each firm in period t, μ_t is the known $J \times 1$ vector of total employment $\mu_{jt} = \sum_{k \in \mathcal{C}_t^j} \ell_{kjt}$ at each firm, and m_t is the unknown $K \times 1$ vector of individuals m_{kt} of each type k. If both S_t and the associated augmented matrix have rank equal to K, then there will be a unique solution which provides m_{kt} for each period t[9]. We can then obtain $\ell_{kjt} = s_{kjt} m_{kt}$ for each firm, type and year.

Given that we have recovered the equilibrium wages and shares, and number of matches, these objects can then be used to recover the model parameters.

D.3. **Identifying the Labor Supply Parameters.** The baseline labor supply equation from the model is

$$\ln \frac{s_{kjt}}{s_{k0t}} = \overline{u}_k + \beta_{1k} \ln \frac{w_{kjt}}{w_{k0t}} + \sum_{g=1}^{G} \tilde{\sigma}_{kg} \ln s_{kj|gt} \mathbb{1}_{j|g} + \ln u_{kjt} \tag{D.16}$$

where $\tilde{\sigma}_{kg} \equiv (1 - 1/\sigma_{kg})$. Define $\mathbb{1}_{j|g} = 1$ if $j \in g$ and 0 else.

The identification challenge is that both the wage and inside share are potentially correlated with the unobserved amenities and thus endogenous. To address this challenge, we propose and apply in the main text an instrumental variables (IV) strategy which leverages exogenous variation in firm productivity. Here we discuss the application and results from some alternate IV strategies.

One source of instruments relies on strategic interactions between firms in wage-setting. In the presence of strategic interactions, the number and characteristics of other firms in a given labor market can be used as instruments. These so-called "BLP instruments" are very common in the industrial organization literature in the context of the product market where the characteristics and number of competing products are used as instruments for prices (see Berry, Levinsohn and Pakes, 1995 (BLP) for the canonical example). In a labor market context, possible BLP instruments might include the number of firms, average size, or average value-added per worker of other firms in the labor market. Azar, Berry and Marinescu (2022a) use the number of vacancies and log employment of competing firms as instruments for advertised wages on a job posting website. In results not reported, we consider the available BLP instruments in our data, such as the number of firms in the same market, and found that they were not sufficiently strong. Thus, we do not emphasize BLP instruments in our setting.

A second source of wage instruments exploits "uniform wage-setting" whereby firms set wages similarly across local labor markets (Hazell et al., 2022). As suggested by Azar,

[9]This is the Rouché-Capelli theorem.

Berry and Marinescu (2022a), this implies that the wage a firm pays in a given market may be driven by the labor market conditions that same firm faces in other markets. We thus considered Hausman instruments for w_{kjg} in market g using the average predicted wage across all markets that firm operates in other than g[10]. In results not reported, we implemented this approach, following Azar, Berry and Marinescu (2022a), but generally found that these instruments were too weak in our setting.

Finally, we considered a shift-share IV approach following Hummels et al. (2014) and Garin and Silvério (2023) to estimate labor supply. To construct this instrument, we rely on firm-product-country level yearly foreign trade data from Statistics Denmark register UHDI and bilateral trade flows from the BACI dataset. We find that our labor supply parameters are comparable to our main estimates reported in Table F3. We do not emphasize these estimates as much in the paper since we are only able to construct the instrument for the small share of the firms in our sample who export. These results are available upon request.

D.4. **Multi-Equation GMM Approach to Estimating Production Parameters.** Estimating equation (5.8) is not straightforward. We cannot use an equation-by-equation approach as we do for the labor supply equation due to the presence of common parameters across equations. While there are only $K+1$ parameters to estimate ($\rho_k \, \forall \, k$ and δ), there are $K \times (K-1)/2$ equations which could be used to estimate the parameters, with no obvious guidance on which to use. Since not all firms employ every labor type, any subset of equations will somewhat arbitrarily ignore the contribution of some firms. If all firms employed some base type of labor, all the labor ratio equations could be cast in terms of that type. However this is not the case, so an alternative is to use all $K \times (K-1)/2$ equations in a multi-equation GMM estimator. Another possible approach would be to treat the multi-equation GMM system nonlinearly and estimate the $K+1$ parameters directly. This would require $K+1$ instruments, for which the obvious choices are lagged labor and wages for each labor type. However, due to the size of the problem this may be intractable.

The approach we take is to treat the system as a set of linear equations with cross-equation parameter restrictions, estimating the compound parameters—such as $\delta(\rho_k - 1)$—and then calculating the structural parameters post-estimation. This has the advantage of being much faster, and also allows specification testing of the model assumptions—since we can test if our estimates of $\delta(\rho_k - 1)$ equal the product of our estimates of δ and $(\rho_k - 1)$. Functionally, we estimate $K \times (K-1)/2$ equations, where each equation (for all a, b in the set of labor

[10]We also exclude markets in the same municipality or industry as g.

types) takes the following form:

$$\begin{aligned} d_{kjt}d_{hjt}\log\frac{\tilde{w}_{ajt}}{\tilde{w}_{bjt}} &= \sum_k \mathbb{1}_{k=a}d_{kjt}\big[\beta_k^1\log\ell_{kjt}-\beta_k^2\log\ell_{kjt-1}\big] \\ &\quad -\sum_h \mathbb{1}_{h=b}d_{hjt}\big[\beta_h^1\log\ell_{hjt}-\beta_h^2\log\ell_{hjt-1}\big] \\ &\quad +\sum_{k,h,t}\mathbb{1}_{k=a}\mathbb{1}_{h=b}d_{kjt}d_{hjt}\Big[\delta\log\frac{\tilde{w}_{kjt-1}}{\tilde{w}_{hjt-1}}+c_{kht}\Big]+\eta_{abjt} \end{aligned} \tag{D.17}$$

where $\beta_k^1 \equiv (\rho_k - 1)$, $\beta_k^2 \equiv \delta(\rho_k - 1)$, and d_{kjt} is an indicator variable which equals 1 if firm j employs labor type k in periods t and $t-1$. This is similar to a "multivariate" regression where all the same regressors appear on the RHS of every equation. We now have $2K+1$ parameters to estimate, and thus need $2K+1$ instruments. Here we use lagged labor ℓ_{kjt-1}, lagged wages w_{kjt-1}, plus squares of both, giving us an overidentified system which we estimate using linear GMM (essentially 2SLS). Note that this approach allows for arbitrary cross-equation patterns of correlation between the error terms η_{abjt}.

D.5. **Passthrough of Productivity Shocks.** Following Lamadon, Mogstad and Setzler (2022) and our own estimation strategy, we regress long changes in average establishment-level log wages by k-group over long changes in log firm-level value added per worker ($VAPW_{jt}$), instrumented by short changes in VAPW. Our empirical strategy follows Morelli and Herkenhoff (2025) by interacting the VAPW shock with both the within-market share and the national share. This also extends Berger, Herkenhoff and Mongey (2022) who conduct a similar exercise but only consider market-level oligopoly.

We use the estimation dataset described in Section 6.1 and Online Appendix E. Table D1 column (2), presents the results of the following regression:

$$\begin{aligned} \Delta_{e,e'}\ln w_{kjt} &= \alpha_0+\alpha_1\Delta_{e,e'}\ln VAPW_{jt}+\alpha_2 s_{kj|gt-3}+\Delta_{e,e'}\ln VAPW_{jt}\times\alpha_3 s_{kj|gt-3} \\ &\quad +\alpha_4 s_{kj|t-3}+\alpha_5\Delta_{e,e'}\ln VAPW_{jt}\times s_{kj|t-3} \end{aligned}$$

where we set $e=2$ and $e'=3$ and the market shares are expressed in percentages. In column (1), we show results of a specification not including market shares. Average k type worker wages go up by 7.2 percent after a 10 percent increase in VAPW.[11] In column (3), we add controls for establishment size, and dummies for firm id, k-group, year, and local labor market. Our estimates indicate that establishments with a relatively larger market share, either local or national, have a relatively lower passthrough rate, consistent with the findings of Morelli and Herkenhoff (2025). This suggests the presence of strategic interactions both at the market and national levels in Denmark.

[11] Running the same specification with establishment-level data rather than establishment-k-group-level data results in a passthrough of 15.6 percent, comparable to the market passthrough estimates for U.S. data from Lamadon, Mogstad and Setzler (2022).

Dependent Variable:	$\Delta_{e,e'} \ln w_{kjt}$ (1)	$\Delta_{e,e'} \ln w_{kjt}$ (2)	$\Delta_{e,e'} \ln w_{kjt}$ (3)
$\Delta_{e,e'} \ln VAPW_{jt}$	0.072***	0.084***	0.067***
	(0.005)	(0.006)	(0.009)
$s_{kj\|gt-3}$		0.000***	0.000***
		(0.000)	(0.000)
$\Delta_{e,e'} \ln VAPW_{jt} \times s_{kj\|gt-3}$		-0.001***	-0.000
		(0.001)	(0.001)
$s_{kj\|t-3}$		0.064***	0.132***
		(0.013)	(0.014)
$\Delta_{e,e'} \ln VAPW_{jt} \times s_{kj\|t-3}$		-0.699***	-0.536***
		(0.145)	(0.126)
Constant	-0.015***	-0.017***	
	(0.000)	(0.000)	
Establishment size	N	N	Y
Firm id FE	N	N	Y
k-group FE	N	N	Y
Year t FE	N	N	Y
g (commuting zone$\times$industry) FE	N	N	Y
Observations	1,093,731	1,093,731	1,093,731

Table D1. Regression of establishment-level long changes in type k average log wages on firm-level long changes in value added per worker on the 3-period lag of the establishment's local labor market share of type-k workers (in percentages) and its interaction with long changes in value added, on the lag of the establishment's national labor market share of type-k workers (in percentages) and its interaction with long changes in value added (2-3). We instrument long changes in log value added with short changes (1-period) in value added per worker. We add controls for the log of establishment size, firm fixed effects, worker type fixed effects, year fixed effects, local labor market fixed effects (3). Robust standard errors in parentheses. Estimation dataset described in Online Appendix E; we drop observations with missing value added data and singleton observations.

Appendix E. Data and Sample Description

Our data consists of several administrative registers provided by Statistics Denmark for the years 2001-2019. These include annual cross-section data from the Danish register-based, matched employer-employee dataset IDA (Integrated Database for Labor Market Research) and other annual datasets, divided into IDAN, IDAS, and IDAP. The datasets are linked by individual identifiers for persons, firms, and establishments. Table E1 lists the relevant datasets and details.

We restrict the dataset to individuals between 26 and 60 years of age who work full-time as employees in the private sector whose job is linked to a physical establishment. We drop individuals employed in the financial sector; firms in the financial sector are not required to report revenue data and very few do. Details on data and sample selection are in Table E2. In total, our dataset consists of $12,742,746$ individual-year combinations. Our sample construction selects the data in a few important ways: The full population of salaried jobs in Denmark in 2001-2019 is 49.3 percent female. This goes down to 35.8 percent when

Category	**Register**	**Variables**
workers	IDAN (jobs yearly panel)	firm and establishment indicator, establishment location, yearly earnings, hours worked, share of the year worked, type of job (primary, secondary), type of job (part-time/full-time), type of job (occupation, DISCO code)
not employed	BEF (population register) IDAN	We classify as not employed all individuals in the relevant age groups who are not recorded in IDAN.
unemployed	IND (income dataset, individual yearly panel), IDAP (worker dataset, individual yearly panel)	unemployment benefits, duration of unemployment
firms and establishments	FIRM, IDAS (workplace panel)	firm revenue and value added, sector of industry (5-digit industry classification based on NACE rev. 2), establishment location (municipality)
k-groups	UDDA (education panel), BEF (individual yearly panel)	age, highest acquired education, gender
commuting zones	Eckert, Hejlesen and Walsh (2022) (available on Fabian Eckert website)	commuting zone (link to municipality)

Table E1. Data Description (Datasets and Variables).

	step	observations	share in public sector	share in financial sector	share full-time	share female	age	avg. yearly earnings (2022 USD)
1	All salaried jobs in Denmark in 2001-2019	76,869,608						
2	Keep jobs held by workers in k-groups	50,263,511	0.229	0.024	0.437	0.493	42.5	42,867
3	Keep jobs with market information	32,486,151	0.355	0.037	0.648	0.487	43	56,389
4	Drop workers in small commuting zones	32,106,644	0.354	0.037	0.768	0.487	43	56,474
5	Drop jobs with no earnings or hours	32,094,227	0.354	0.037	0.648	0.487	43	56,493
6	Drop public sector jobs	20,719,775		0.057	0.660	0.358	42.5	59,641
7	Drop financial sector jobs	19,538,794			0.653	0.349	42.4	58,296
8	Keep full-time, highest-paying jobs	**12,742,741**				0.318	43.5	71,491
9	Keep only period 2004-2016	8,614,259						

Table E2. Worker Sample Selection.

we drop the public sector and further to 31.8 percent when we exclude the financial sector and non-full-time jobs. Workers in the private-sector with full-time jobs are on average one year older than the full worker population, and have average yearly earnings of $71,491$ USD, higher than the full-worker-population average of $42,867$ USD.

Find a detailed description of the selection steps below:

(1) This step excludes self employed and employers, and their spouses if their main source of income is from assisting the spouse's enterprise; it includes all other types of jobs.
(2) This step drops workers not appearing in the population registers, younger and older workers, as well as workers with no education status recorded (this applies mostly to immigrant workers). Therefore, this step excludes jobs held by workers not resident in Denmark.
(3) This step drops jobs without real establishment code, i.e., all non-primary jobs and primary jobs with missing or fictitious establishment code. Primary jobs are the most important connection to the labor market (longest employment period and largest ATP payments). Workers with fictitious workplaces (establishment nr. = 0) are those who cannot be linked to any of the employer's registered workplaces, either because they work from home or in various workplaces (such as cleaners, home nurses). Workers with no workplace (establishment nr. = .) are those with multiple workplaces for which one unique workplace cannot be identified. In 2,491,168 instances, where the establishment information is missing only in one year during a continuous employment spell at the same firm, we impute it.
(4) Drop jobs in establishments in Christiansœ, Bornholm, Samsœ, and Æro.
(5) Drop jobs with no information on earnings or hours
(6) Drop if the sector of industry of the employer is one of the following 1-digit NACE rev.2 codes {O,P,Q,T,U,X}.
(7) Drop if the sector of industry of the employer is nacee-2 code K (this sector has an extreme underreporting of revenue data).
(8) We define full-time jobs as jobs with weekly schedule of 30 hours or more.

We denote establishments with the subscript j, time (years) with the subscript t, and worker type (k-groups) with the subscript k. k-groups are divided by gender (male or female) age group (26-35, 36-50, 51-60) and education level (completed or not tertiary education). We define a local labor market g as a commuting zone and industry pairing. We use the 3-digit EU industry classification NACE Rev. 2 (Carré, 2008) and we drop the public and financial sectors. We use 16 of the 23 commuting zones computed for 2005 by Eckert, Hejlesen and Walsh (2022) using the Tolbert and Sizer (1996) method for Denmark. We drop six of the commuting zones that are small islands relatively separated from the mainland (Christiansœ, Bornholm, Samsœ, and Æro), and we merge the two North Jutland commuting zones of Aalborg and Frederikshavn. In our final estimation dataset, we have 2,757 local labor markets. We collapse the individual-level dataset at the (k, j, t) level leading to $4,487,628$ observations. We restrict the estimation dataset to only establishments with no missing values for any of the key variables. Table E3 details the sample selection process.

	step	total observations	unique establishments
1	collapse at the k-group-establishment-year (k, j, t) level	4,487,620	259,190
2	merge revenue data (firm, year)	-	-
3	add share of non-employed/unemployed and average unemployment income	-	-
4	drop observations with wage bill to revenue ratio above 80 percent (drops all observations with missing revenue)	4,054,229	238,295
	keep observations for firms that appear at least once in the estimation dataset	**3,069,502**	**63,526**
5	create estimation variables	-	-
6	keep observations in 2004-2017 to accommodate for long run lags $(x_{jkt+2} - x_{jkt-3})$ and data break	2,332,058	-
7	drop firms/k-groups with not enough longevity to allow for computing short-run lags $(x_{jkt} - x_{jkt-1})$	2,294,908	-
8	drop firms/k-groups with not enough longevity to allow for computing long-run lags $(x_{jkt+2} - x_{jkt-3})$	1,983,593	-
9	drop firms employing only one k-group (necessary for the second instrument)	**1,101,543**	**63,526**

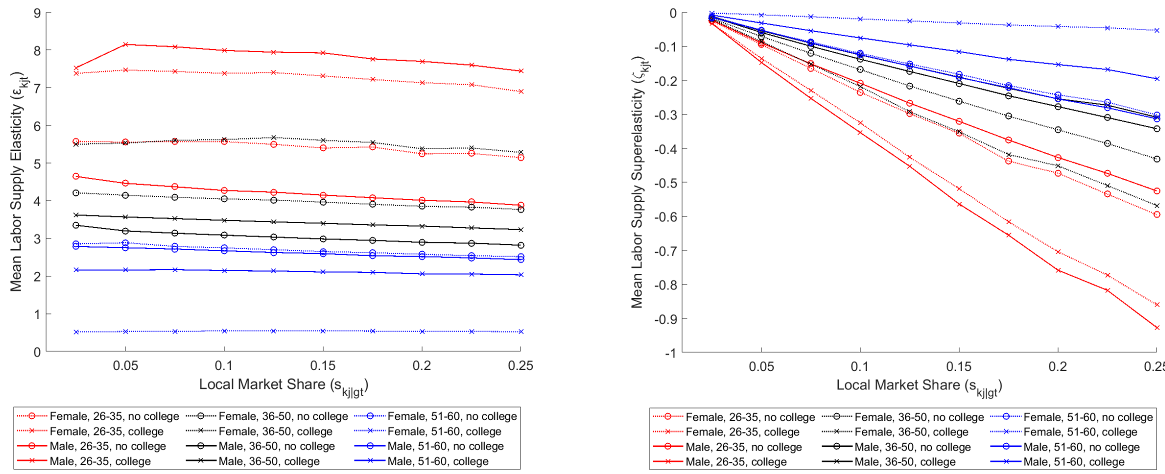
Table E3. Establishment Sample Selection and Construction of the Estimation Dataset. Start with panel of selected workers in years 2001-2019. Variables: full-time-equivalent, earnings, k-group (gender, age, education), local market (commuting zone, industry), firm, establishment, year (12,742,741 individuals).

We measure labor inputs in terms of full-time equivalents (FTE). We calculate the full-time equivalent as the number of hours worked in the calendar year divided by the average number of full-time hours worked by full-time workers in Denmark over the same period, where we define a full-time worker as an individual who works 30+ hours a week. This implies that if an individual works full-time in one establishment for six months, she will be counted as half of a FTE. We use non-employment (unemployment + non-participation) as the outside option. We define non-participation as an individual not observed in the linked employer-employee data for a (part of the) year. Non-participation income is set to zero. Unemployment spells and unemployment income are observed directly in the data. Therefore, non-employment income consists of unemployment income for the unemployed workers. This includes cash assistance, unemployment benefits, leave benefits, and other assistance benefits, but—similarly to our measure of wages—it does not include long-term sickness or pension benefits.

The key variables we use in the estimation are:

- w_{kjt}: mean earnings by k-group, establishment, year
- w_{k0t}: mean non-employment income by k-group, year
- s_{kjt} and $s_{kj|gt}$: employment shares, by k-group, establishment, year, overall and by market g (inside shares)
- s_{0t}: overall non-employment shares, by k-group, establishment, year (calculated by summing the non-employment spells at the k level and dividing by the total number of FTEs and non-employment spells in the data)
- $s_{\sim kj|gt}$: sum of the inside shares for all other labor types employed by establishment j, by k-group, year, market
- R_{jt}: establishment-level revenue by year, obtained allocating firm revenue across establishments in proportion to their wage bills

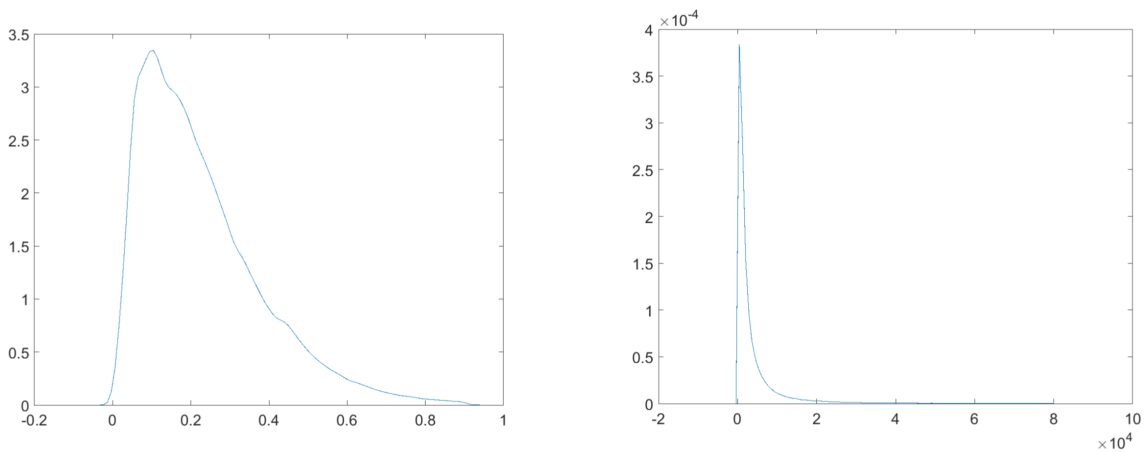
Appendix F. Appendix Figures and Tables



(A) labor supply elasticity ($\mathcal{E}_{kjt}$)

(B) cross-wage super-elasticity (ζ_{kjt})

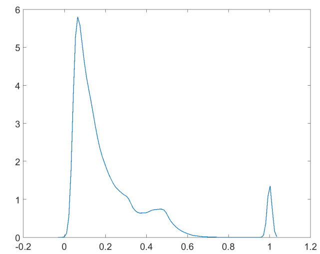
Figure F1. Labor supply elasticities by local market share and worker type. Local market: Commuting Zone×Industry. Panel (a) plots average estimated labor supply elasticities ($\mathcal{E}_{kjt}$) over the local market share ($s_{kj|gt}$), by k-group. Panel (b) plots average estimated labor supply super-elasticities (ζ_{kjt}) over the local market share ($s_{kj|gt}$), by k-group. Establishment-level elasticities are averaged across years and local markets by the establishment local market share bin (10 bins between 0 and 0.25).



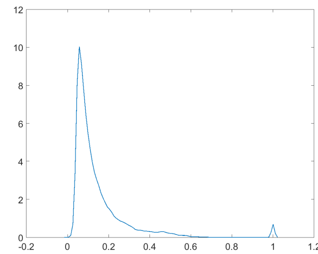
(A) Distribution of α_{jt}

(B) Distribution of $\tilde{\theta}_{jt}^{\alpha_{jt}}$

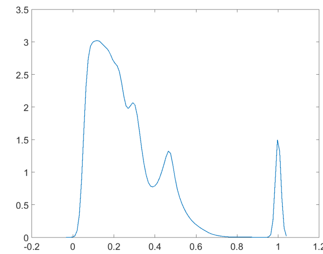
Figure F2. Panel (a) shows the distribution of the scale parameter α_{jt} (equation (5.10)). The mean of this distribution is 0.214 and the median is 0.181. Panel (b) shows the distribution of productivity term $\tilde{\theta}_{jt}^{\alpha_{jt}}$, truncated at the 99th percentile (equation (5.11)). The mean of the truncated distribution is 6,538 (in 2021 thousands of Danish krona). The 90-10 ratio for $\tilde{\theta}_{jt}^{\alpha_{jt}}$ over all private sector firms in the economy is 22.8.



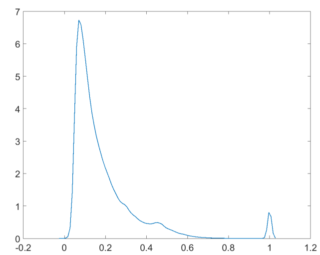
(A) Distribution of γ_{1jt}



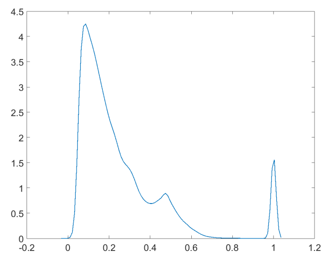
(B) Distribution of γ_{2jt}



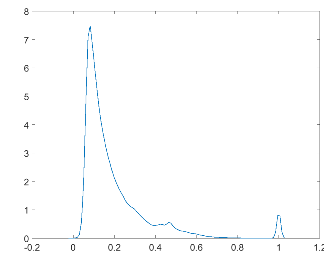
(C) Distribution of γ_{3jt}



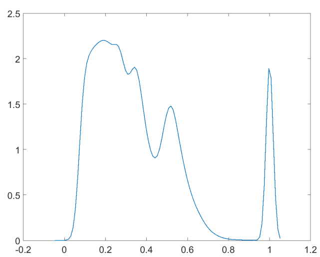
(D) Distribution of γ_{4jt}



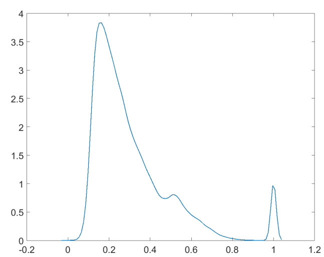
(E) Distribution of γ_{5jt}



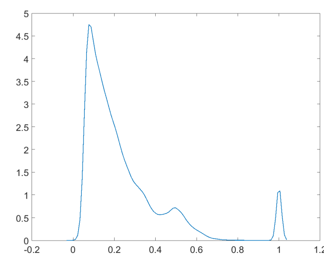
(F) Distribution of γ_{6jt}



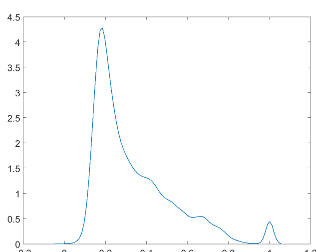
(G) Distribution of γ_{7jt}



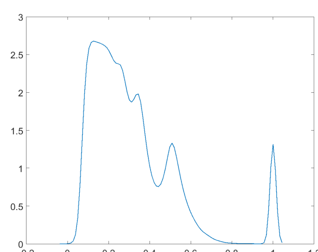
(H) Distribution of γ_{8jt}



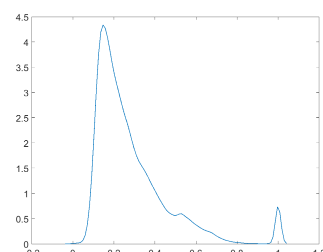
(I) Distribution of γ_{9jt}



(J) Distribution of γ_{10jt}

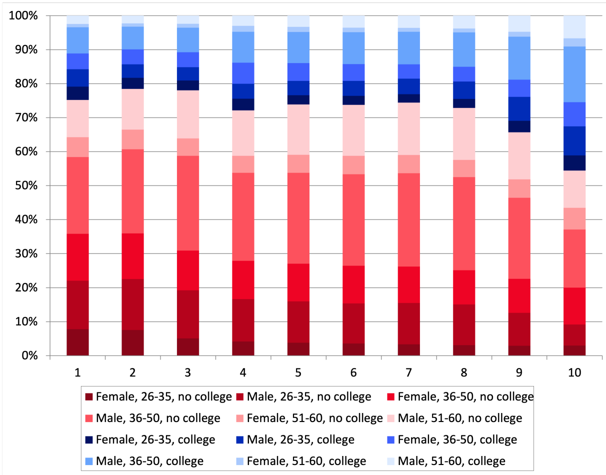


(K) Distribution of γ_{11jt}

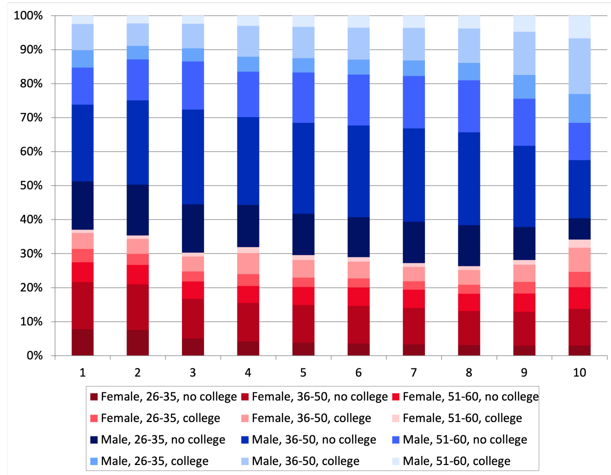


(L) Distribution of γ_{12jt}

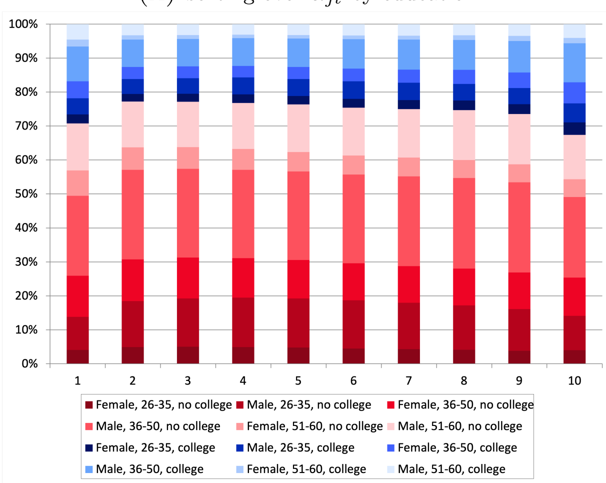
FIGURE F3. The 12 panels show the distribution of the normalized productivity parameter γ_{kjt} for each of the 12 k-groups (equation (5.9)).



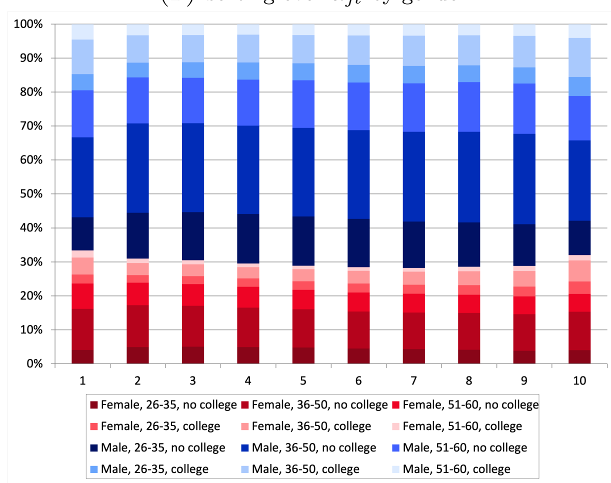
(A) sorting over α_{jt} by education



(B) sorting over α_{jt} by gender



(C) sorting over $\tilde{\theta}_{jt}^{\alpha_{jt}}$ by education



(D) sorting over $\tilde{\theta}_{jt}^{\alpha_{jt}}$ by gender

FIGURE F4. Sorting of worker types across deciles of the distribution of two separate components of the establishment wage premium: returns to scale α_{jt} and total factor productivity $\tilde{\theta}_{jt}^{\alpha_{jt}}$. This figure shows the employment share of each k-group for each deciles of the establishment-level distribution of α_{jt} and $\tilde{\theta}_{jt}^{\alpha_{jt}}$. In Panels (a) and (c), the k-groups are ordered by education: non-college graduates in red (older workers in lighter red) and college graduates in blue (older workers in lighter blue). In Panels (a) and (c), the k-groups are ordered by gender: women in red (college educated in lighter red) and men in blue (college educated in lighter blue).

	n. unique estab.	n. estab. per firm		n. of workers per estab.		n. of k-groups per estab.		estab. revenue (1,000 UDS)		average wage (USD)		local market share $s_{kj\|gt}$	
commuting zone		mean	st. dev.	mean	st. dev.	mean	st. dev.	mean	st. dev.	mean	st. dev.	mean	st. dev.
1. North and East Zealand (Copenhagen)	92,731	1.232	3.814	8.454	40.626	2.685	2.356	6,171	61,361	65,148	36,819	0.018	0.071
2. West and South Zealand (Slagelse)	10,718	1.241	4.367	5.816	33.274	2.348	1.897	3,941	55,333	55,326	15,962	0.119	0.214
3. West and South Zealand (Køge)	11,958	1.205	3.241	5.715	19.334	2.383	1.929	3,533	22,430	55,888	17,122	0.112	0.210
4. West and South Zealand (Nykøbing Falster)	4,431	1.249	4.230	5.293	14.390	2.316	1.794	2,729	10,837	51,731	13,878	0.205	0.288
7. Fyn (Odense)	18,879	1.254	3.459	7.285	24.103	2.686	2.251	4,829	29,983	56,571	26,792	0.073	0.160
8. Fyn (Svendborg)	2,934	1.175	3.061	4.953	9.919	2.400	1.917	2,820	8,953	54,654	17,221	0.275	0.317
9. South Jutland (Sønderborg)	5,720	1.243	3.210	8.191	48.529	2.614	2.162	5,614	31,401	54,918	16,683	0.172	0.262
10. South Jutland (Ribe)	2,053	1.183	2.188	5.554	17.850	2.298	1.879	4,179	22,627	52,262	13,968	0.298	0.334
11. South Jutland (Kolding)	9,611	1.254	2.677	7.322	19.109	2.727	2.280	4,924	17,371	56,778	17,735	0.123	0.219
12. Mid-South Jutland (Vejle)	14,565	1.194	2.730	7.820	45.272	2.680	2.258	6,017	58,046	57,835	21,745	0.088	0.181
13. South-West Jutland (Esbjerg)	10,561	1.226	3.130	6.981	22.509	2.590	2.167	5,420	58,499	55,862	16,837	0.111	0.208
14. West Jutland (Herning)	9,517	1.200	3.119	7.040	22.462	2.605	2.156	4,583	22,913	55,664	15,332	0.115	0.208
15. North-West Jutland (Thisted)	2,138	1.207	2.728	6.329	21.196	2.416	1.975	4,009	15,606	54,166	13,972	0.292	0.326
16. East Jutland (Aarhus)	31,814	1.245	3.775	7.399	24.617	2.678	2.271	5,160	53,258	59,102	22,934	0.047	0.123
17. Mid-North Jutland (Viborg)	7,980	1.188	3.828	6.901	47.707	2.493	2.077	4,071	26,117	54,906	15,959	0.137	0.233
19. North Jutland (Aalborg)	23,580	1.208	2.551	6.523	21.000	2.520	2.115	4,499	49,905	55,542	18,251	0.062	0.147
industry													
A. Agriculture, forestry, and fishery	13,499	1.042	0.746	2.302	4.045	1.643	1.246	1,720	2,909	48,810	13,767	0.110	0.206
B. Mining and quarrying	431	1.756	3.568	13.872	62.902	2.767	2.500	35,220	298,848	72,556	101,517	0.315	0.332
C. Manufacturing	20,892	1.174	1.363	18.924	73.662	3.872	2.978	12,355	73,817	60,794	18,468	0.177	0.282
D. Electricity, gas, steam etc.	921	1.263	1.592	15.340	46.976	3.372	2.926	34,650	321,543	73,488	30,898	0.222	0.288
E. Water supply, sewerage etc.	1,954	2.123	3.517	10.479	21.034	3.112	2.306	4,353	14,119	59,114	13,886	0.214	0.296
F. Construction	31,942	1.050	0.708	5.145	14.408	2.298	1.696	2,649	12,075	57,610	14,378	0.032	0.103
G. Wholesale and retail trade	69,175	1.383	5.732	5.514	15.559	2.518	1.992	6,680	36,576	56,683	21,619	0.039	0.106
H. Transportation	15,580	1.270	5.066	11.277	50.020	2.794	2.331	7,665	114,425	57,890	25,777	0.059	0.156
I. Accommodation and food services	15,791	1.236	2.937	3.370	9.242	2.038	1.638	1,488	4,217	48,049	13,443	0.089	0.182
J. Information and communication	15,523	1.174	2.949	10.968	49.839	2.912	2.604	5,163	29,492	76,131	40,250	0.067	0.183
L. Real estate	13,051	1.343	2.305	3.541	8.919	2.080	1.728	1,139	4,436	59,727	25,909	0.045	0.111
M. Knowledge-based services	27,529	1.139	1.298	7.590	30.831	2.753	2.433	2,798	18,008	72,659	47,190	0.070	0.159
N. Travel agent, cleaning etc.	13,777	1.279	2.242	6.724	19.534	2.668	2.325	3,153	12,084	59,338	36,342	0.138	0.240
R. Arts, entertainment, recreation	5,790	1.423	3.026	5.765	14.060	2.799	2.420	1,048	22,416	54,942	19,372	0.099	0.193
S. Other services	13,335	1.126	1.543	4.523	13.985	2.222	1.972	419	2,547	55,563	16,467	0.068	0.143
All local labor markets	259,190	1.227	3.494	7.414	33.071	2.611	2.223	5,199	49,993	59,311	27,048	0.074	0.175

TABLE F1. Establishment characteristics, full sample, all years. Source: Administrative registers, Statistics Denmark. Full population of private sector establishments in Denmark (step 1 in Table E3). Commuting zones computed for 2005 by Eckert, Hejlesen and Walsh (2022), largest city in parentheses. We drop six small islands and we merge Aalborg and Frederikshavn. 1-digit industry classification based on NACE rev. 2. We exclude the public sector, including the health and education sectors. Revenue and average wage at the firm in 2022 USD. Local labor market shares calculated as average share of each k-group workers employed at the establishment over total number of k-group workers in the commuting zone×3-digit industry×year market g.

	n. unique estab.	n. estab. per firm		n. of workers per estab.		n. of k-groups per estab.		estab. revenue (1,000 UDS)		average wage (USD)		local market share $s_{kj\|gt}$	
commuting zone		mean	st. dev.	mean	st. dev.	mean	st. dev.	mean	st. dev.	mean	st. dev.	mean	st. dev.
1. North and East Zealand (Copenhagen)	20,352	1.203	2.905	13.603	54.919	3.672	2.581	11,148	71,714	68,274	24,494	0.021	0.078
2. West and South Zealand (Slagelse)	2,594	1.198	2.172	9.008	46.099	3.106	2.087	6,915	77,429	57,408	13,925	0.123	0.217
3. West and South Zealand (Køge)	2,823	1.256	3.961	9.000	26.336	3.212	2.113	6,138	31,361	58,367	15,116	0.119	0.217
4. West and South Zealand (Nykøbing Falster)	1,095	1.234	3.769	7.981	18.888	3.028	1.954	4,528	14,376	53,727	12,963	0.212	0.288
7. Fyn (Odense)	4,923	1.254	3.575	11.125	31.529	3.575	2.438	7,969	39,127	58,870	16,999	0.080	0.169
8. Fyn (Svendborg)	745	1.280	6.141	7.356	12.324	3.189	2.086	4,691	11,938	56,857	14,928	0.281	0.320
9. South Jutland (Sønderborg)	1,566	1.249	4.162	12.882	65.503	3.433	2.352	9,352	41,824	57,073	14,425	0.172	0.257
10. South Jutland (Ribe)	510	1.141	1.471	9.010	24.495	3.129	2.118	7,210	31,333	54,685	12,547	0.327	0.336
11. South Jutland (Kolding)	2,637	1.225	1.760	11.245	24.572	3.613	2.485	8,000	22,656	59,639	16,070	0.124	0.217
12. Mid-South Jutland (Vejle)	3,958	1.222	3.855	12.182	60.543	3.587	2.456	10,021	78,260	60,381	18,023	0.092	0.184
13. South-West Jutland (Esbjerg)	2,899	1.227	2.530	10.648	28.233	3.445	2.362	8,952	79,034	58,512	15,138	0.116	0.212
14. West Jutland (Herning)	2,654	1.198	2.821	10.817	29.165	3.464	2.344	7,453	30,365	57,933	13,439	0.119	0.210
15. North-West Jutland (Thisted)	587	1.218	3.812	9.958	28.068	3.217	2.174	6,650	20,664	56,651	12,735	0.299	0.330
16. East Jutland (Aarhus)	8,199	1.241	3.929	11.303	31.179	3.588	2.456	8,625	72,640	61,478	16,908	0.051	0.127
17. Mid-North Jutland (Viborg)	2,077	1.143	2.313	10.737	65.201	3.349	2.254	6,614	28,985	57,435	15,628	0.147	0.241
19. North Jutland (Aalborg)	5,907	1.224	2.994	10.202	27.552	3.412	2.330	7,560	67,593	57,936	16,245	0.066	0.154
industry													
A. Agriculture, forestry, and fishery	2,235	1.047	0.600	3.782	5.723	2.372	1.583	2,840	4,262	50,896	11,757	0.122	0.213
B. Mining and quarrying	140	1.555	2.314	17.892	68.180	3.553	2.662	48,421	357,241	70,750	31,235	0.327	0.333
C. Manufacturing	8,902	1.166	1.151	24.457	84.925	4.632	2.998	16,004	84,927	61,765	14,128	0.178	0.277
D. Electricity, gas, steam etc.	306	1.180	1.064	20.179	57.205	4.095	3.001	64,549	456,296	74,386	33,991	0.218	0.269
E. Water supply, sewerage etc.	431	1.553	1.876	12.983	25.803	3.719	2.517	7,940	19,440	60,679	12,300	0.248	0.301
F. Construction	8,742	1.060	0.812	7.549	17.836	3.005	1.814	3,949	15,334	59,681	12,588	0.035	0.102
G. Wholesale and retail trade	21,217	1.357	4.895	7.916	19.278	3.254	2.156	9,794	45,456	59,433	19,340	0.042	0.108
H. Transportation	4,290	1.320	5.428	16.608	61.499	3.644	2.484	11,220	105,958	59,758	18,708	0.066	0.161
I. Accommodation and food services	2,016	1.210	1.993	5.750	14.367	3.018	2.008	2,709	6,364	51,705	12,648	0.092	0.182
J. Information and communication	3,521	1.212	2.792	18.323	63.986	4.175	2.851	8,926	38,975	76,905	24,703	0.074	0.191
L. Real estate	1,610	1.175	1.128	5.155	12.528	2.922	2.013	2,714	7,538	68,106	28,961	0.055	0.115
M. Knowledge-based services	6,116	1.160	1.109	12.136	40.652	3.912	2.622	4,899	24,755	72,717	24,334	0.072	0.154
N. Travel agent, cleaning etc.	2,697	1.185	1.206	7.918	23.035	3.305	2.284	5,492	17,011	62,117	20,404	0.146	0.244
R. Arts, entertainment, recreation	389	1.085	0.698	9.414	20.773	3.896	2.719	8,397	70,779	60,055	17,245	0.159	0.265
S. Other services	914	1.166	3.365	7.322	16.345	2.951	2.251	2,260	5,253	57,559	16,769	0.109	0.197
All local labor markets	63,526	1.219	3.240	11.591	44.040	3.515	2.427	8,909	62,960	61,787	19,573	0.080	0.180

TABLE F2. Establishment characteristics, by commuting zone, estimation sample, all years. Source: Administrative registers, Statistics Denmark. Restricted sample of establishments with no missing values for the key estimation variables (step 5 in Table E3). Commuting zones computed for 2005 by Eckert, Hejlesen and Walsh (2022), largest city in parentheses. We drop six small islands and we merge Aalborg and Frederikshavn. 1-digit industry classification based on NACE rev. 2. We exclude the public sector, including the health and education sectors. Revenue and average wage at the firm in 2022 USD. Local labor market shares calculated as average share of each k-group workers employed at the establishment over total number of k-group workers in the commuting zone×3-digit industry×year market g.

		IV			OLS		
		β_k	σ_{kg}		β_k	σ_{kg}	
	k-group (k)		CZ 1 (CPH)	Avg. across CZ		CZ 1 (CPH)	Avg. across CZ
1	Female, 26-35, no college	2.977	1.854	1.911	-0.024	2.314	2.117
		[2.216; 3.487]	[1.635; 2.147]		[-0.047; -0.000]	[2.117; 2.413]	
2	Female, 26-35, college	3.806	1.958	2.216	-0.094	2.245	1.831
		[2.317; 4.507]	[1.602; 2.254]		[-0.128; -0.058]	[2.005; 2.355]	
3	Male, 26-35, no college	1.985	2.345	2.292	0.331	2.673	2.285
		[1.871; 2.210]	[2.049; 2.534]		[0.317; 0.339]	[2.440; 2.697]	
4	Male, 26-35, college	4.111	1.697	1.549	0.323	2.004	1.729
		[3.015; 4.620]	[1.527; 1.895]		[0.301; 0.347]	[1.871; 2.053]	
5	Female, 36-50, no college	2.184	1.957	1.929	0.215	2.160	1.920
		[1.903; 2.487]	[1.796; 2.075]		[0.203; 0.225]	[1.990; 2.173]	
6	Female, 36-50, college	3.214	1.704	2.137	0.000	1.842	1.693
		[2.003; 3.573]	[1.526; 1.828]		[0.046; 0.082]	[1.702; 1.887]	
7	Male, 36-50, no college	1.565	2.215	2.025	0.271	2.323	2.108
		[1.514; 1.744]	[1.982; 2.251]		[0.262; 0.275]	[2.134; 2.299]	
8	Male, 36-50, college	2.090	1.748	1.714	0.106	1.840	1.593
		[1.691; 2.333]	[1.611; 1.818]		[0.094; 0.118]	[1.707; 1.841]	
9	Female, 51-60, no college	1.415	1.898	1.926	0.234	2.125	1.938
		[0.751; 1.953]	[1.723; 2.093]		[0.221; 0.244]	[1.980; 2.223]	
10	Female, 51-60, college	0.331	1.544	1.829	0.194	1.675	1.594
		[-0.942; 1.559]	[1.414; 1.710]		[0.168; 0.223]	[1.555; 1.781]	
11	Male, 51-60, no college	1.299	2.103	2.148	0.260	2.245	2.165
		[1.119; 1.458]	[1.879; 2.191]		[0.249; 0.266]	[2.054; 2.289]	
12	Male, 51-60, college	1.316	1.640	1.665	0.173	1.763	1.633
		[0.588; 1.746]	[1.528; 1.768]		[0.154; 0.187]	[1.654; 1.827]	

TABLE F3. Parameter estimates for equation (5.4), OLS and IV. We estimate the parameters separately by k-group. The first column are the point estimates for β_k. The second column shows estimates for the σ_{kg} for the Copenhagen metro area). The third column shows the average σ_{kg} estimate across commuting zones. Bootstrapped 95 percent confidence intervals in square brackets (Hall, 1992).

Source: Administrative registers, Statistics Denmark.

	$log(u_{kj})$	
Commuting zone (reference: North and East Zealand (Copenhagen))		
West and South Zealand (Slagelse)	-1.042	(0.003)
West and South Zealand (Køge)	-1.150	(0.003)
West and South Zealand (Nykøbing Falster)	-1.552	(0.004)
Fyn (Odense)	-0.817	(0.002)
Fyn (Svendborg)	-1.693	(0.005)
South Jutland (Sønderborg)	-1.215	(0.004)
South Jutland (Ribe)	-2.028	(0.007)
South Jutland (Kolding)	-1.005	(0.003)
Mid-South Jutland (Vejle)	-0.942	(0.002)
South-West Jutland (Esbjerg)	-1.083	(0.003)
West Jutland (Herning)	-1.097	(0.003)
North-West Jutland (Thisted)	-1.686	(0.007)
East Jutland (Aarhus)	-0.471	(0.002)
Mid-North Jutland (Viborg)	-1.159	(0.003)
North Jutland (Aalborg)	-0.501	(0.002)
Industry (reference: A. Agriculture, forestry, and fishery)		
B. Mining and quarrying	-0.664	(0.014)
C. Manufacturing	-0.191	(0.005)
D. Electricity, gas, steam etc.	-0.215	(0.008)
E. Water supply, sewerage etc.	-0.604	(0.008)
F. Construction	0.411	(0.005)
G. Wholesale and retail trade	0.407	(0.004)
H. Transportation	0.326	(0.005)
I. Accommodation and food services	-0.169	(0.005)
J. Information and communication	0.394	(0.005)
L. Real estate	0.090	(0.006)
M. Knowledge-based services	0.235	(0.005)
N. Travel agent, cleaning etc.	-0.404	(0.005)
R. Arts, entertainment, recreation	-0.246	(0.007)
S. Other services	-0.302	(0.007)
Log of establishment size (number of workers)	1.775	(0.004)
Log of establishment wagebill (thousands 2022 USD)	-1.526	(0.004)
Log of establishment revenue (thousands 2022 USD)	-0.019	(0.001)
Log of firm size (number of workers)	0.016	(0.000)
Observations	2,360,853	
R^2	0.803	

Table F4. OLS of estimated deterministic preferences for amenities $log(u_{kj})$ on k-group, commuting zone, industry, and year indicators, and establishment characteristics (logarithm of firm and establishment size in number of workers, and logarithm of establishment wage bill and revenue). We report coefficients for commuting zone, industry, and establishment characteristics. Robust standard errors in parentheses, $p < 0.01$.

		IV			IV	OLS
	k-group	$\rho_k - 1$	$\delta(\rho_k - 1)$	δ	ρ_k	ρ_k
1	Female, 26-35, no college	0.010	0.010	0.803	1.009	0.990
		[-0.000; 0.017]	[0.002; 0.016]	[0.800; 0.805]	[1.000; 1.017]	[0.986; 0.992]
2	Female, 26-35, college	0.030	0.030		1.031	0.989
		[0.020; 0.040]	[0.020; 0.039]		[1.020; 1.040]	[0.986; 0.993]
3	Male, 26-35, no college	0.010	0.010		1.012	0.992
		[0.004; 0.019]	[0.004; 0.016]		[1.004; 1.019]	[0.991; 0.995]
4	Male, 26-35, college	0.032	0.032		1.030	0.985
		[0.019; 0.038]	[0.021; 0.039]		[1.019; 1.038]	[0.982; 0.988]
5	Female, 36-50, no college	0.020	0.020		1.021	0.982
		[0.011; 0.031]	[0.011; 0.029]		[1.011; 1.031]	[0.980; 0.985]
6	Female, 36-50, college	-0.004	-0.004		1.003	0.995
		[-0.011; 0.018]	[-0.017; 0.011]		[0.989; 1.018]	[0.991; 1.000]
7	Male, 36-50, no college	-0.015	-0.015		0.983	0.983
		[-0.026; -0.006]	[-0.024; -0.005]		[0.974; 0.994]	[0.981; 0.985]
8	Male, 36-50, college	-0.066	-0.066		0.936	1.003
		[-0.082; -0.045]	[-0.083; -0.048]		[0.918; 0.955]	[0.999; 1.008]
9	Female, 51-60, no college	0.011	0.011		1.014	1.003
		[0.002; 0.026]	[0.000; 0.023]		[1.002; 1.026]	[0.997; 1.003]
10	Female, 51-60, college	-0.003	-0.003		1.008	1.000
		[-0.019; 0.040]	[-0.029; 0.028]		[0.981; 1.040]	[1.032; 1.048]
11	Male, 51-60, no college	-0.003	-0.001		0.998	1.041
		[-0.015; 0.009]	[-0.013; 0.009]		[0.985; 1.009]	[0.989; 0.995]
12	Male, 51-60, college	-0.041	-0.041		0.964	0.992
		[-0.054; -0.008]	[-0.058; -0.015]		[0.946; 0.992]	[1.024; 1.040]

TABLE F5. Parameter estimates for the production function, IV. The first two columns are the point estimates for $(\rho_k - 1)$ and $\delta(\rho_k - 1)$ from equation (5.8). The third and fourth columns show the implied values for δ and ρ_k. The fifth column shows the OLS estimate for ρ_k. Bootstrapped 95 percent confidence intervals in square brackets.

	k-group	η_{kjt} Mean	Median	P10	P90
1	Female, 26-35, no college	-5.667	-9.276	-77.774	-1.188
2	Female, 26-35, college	2.131	-6.995	-80.058	100.415
3	Male, 26-35, no college	-13.378	-5.582	-25.648	-1.709
4	Male, 26-35, college	-41.663	-8.602	-74.412	78.132
5	Female, 36-50, no college	-49.373	-7.233	-42.374	-1.283
6	Female, 36-50, college	-24.723	-12.799	-58.967	-2.673
7	Male, 36-50, no college	-4.126	-3.051	-7.844	-1.425
8	Male, 36-50, college	-3.925	-4.519	-9.071	-1.969
9	Female, 51-60, no college	-5.878	-9.465	-59.891	-1.654
10	Female, 51-60, college	-43.820	-8.597	-62.749	-1.660
11	Male, 51-60, no college	-7.524	-4.753	-14.639	-1.884
12	Male, 51-60, college	-7.184	-6.976	-15.970	-2.287

TABLE F6. Moments of the firm-level labor demand elasticities $\eta_{kjt} \equiv F_k^j / \ell_{kj} F_{kk}^j$.

k-group		**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
Female, 26-35, no college	**1**	0	-46	-100	-51	-138	107	253	64	-132	-90	2,582	23
Female, 26-35, college	**2**	101	0	-68	-33	-12	-229	164	34	3	24	2,989	33
Male, 26-35, no college	**3**	-72	-41	0	1	-70	-250	-131	-8	-85	-56	730	20
Male, 26-35, college	**4**	-79	-33	-218	0	-57	-108	-210	-1	-68	10	394	24
Female, 36-50, no college	**5**	-21	-36	-134	-24	0	46	-46	15	-52	25	1,160	18
Female, 36-50, college	**6**	-804	-89	54	-99	-605	0	503	77	-553	-698	2,989	63
Male, 36-50, no college	**7**	90	-23	-265	-82	11	-326	0	23	-13	192	476	29
Male, 36-50, college	**8**	166	-27	-349	-62	6	-102	-19	0	26	179	1,177	27
Female, 51-60, no college	**9**	-42	-38	-90	-39	-79	-208	88	22	0	-7	1,276	26
Female, 51-60, college	**10**	-164	-58	-107	-53	-315	329	669	109	-231	0	5,815	60
Male, 51-60, no college	**11**	706	29	-275	-12	331	-592	18	6	280	873	0	37
Male, 51-60, college	**12**	-111	-31	-106	-34	-57	-392	50	16	-78	-66	-42	0

TABLE F7. Each cell is the mean Morishima elasticity of substitution calculated across all firms which employ both types of labor.

k-group	$\mathbb{E}[\log w]$	Var($\log w$)	Emp	GCI	EV	EV-V	EV-G
Female, 26–35, No College	3.4657	0.0484	0.307	0.0460	1.000	–	–
Female, 26–35, College	3.7235	0.0650	0.482	0.0168	1.000	–	–
Male, 26–35, No College	3.6604	0.0525	0.630	0.0036	1.000	–	–
Male, 26–35, College	3.8588	0.0709	0.709	0.0034	1.000	–	–
Female, 36–50, No College	3.6436	0.0537	0.497	0.0085	1.000	–	–
Female, 36–50, College	3.9766	0.0858	0.673	0.0041	1.000	–	–
Male, 36–50, No College	3.8212	0.0626	0.737	0.0015	1.000	–	–
Male, 36–50, College	4.1971	0.1061	0.861	0.0011	1.000	–	–
Female, 51–60, No College	3.6352	0.0603	0.422	0.0166	1.000	–	–
Female, 51–60, College	3.9322	0.1687	0.469	0.0164	1.000	–	–
Male, 51–60, No College	3.8014	0.0712	0.684	0.0023	1.000	–	–
Male, 51–60, College	4.2306	0.1527	0.771	0.0019	1.000	–	–

(A) Baseline

$\mathbb{E}[\log w]$	Var($\log w$)	Emp	GCI	EV	EV-V	EV-G
3.7547	0.1247	0.831	0.0013	5.004	1.039	2.965
4.0707	0.1091	0.959	0.0014	5.373	2.673	1.700
3.7672	0.1213	0.578	0.0042	0.801	-0.124	-0.074
4.2131	0.0993	0.996	0.0010	5.064	3.315	0.750
3.7372	0.1248	0.712	0.0014	1.975	-0.192	1.167
4.1800	0.1330	0.956	0.0007	3.280	1.246	1.034
3.8376	0.1495	0.436	0.0104	0.311	0.041	-0.730
4.2655	0.1865	0.732	0.0014	0.529	-0.391	-0.080
3.6985	0.1336	0.344	0.0267	0.743	0.034	-0.291
3.8928	0.2521	0.018	0.7774	0.033	2.347	-3.313
3.8838	0.1994	0.305	0.0354	0.230	0.338	-1.107
4.4105	0.2737	0.300	0.0426	0.149	0.196	-1.047

(B) CF A: Equal Deterministic Amenities

k-group	$\mathbb{E}[\log w]$	Var($\log w$)	Emp	GCI	EV	EV-V	EV-G
Female, 26–35, No College	3.3622	0.1213	0.056	0.5104	0.285	-0.254	-0.461
Female, 26–35, College	2.7037	2.9818	0.027	0.7299	0.122	-0.465	-0.413
Male, 26–35, No College	3.5762	0.0517	0.665	0.0025	1.170	-0.023	0.193
Male, 26–35, College	2.3218	4.3512	0.022	0.7788	0.065	-0.482	-0.452
Female, 36–50, No College	3.5830	0.0563	0.374	0.0235	0.571	-0.074	-0.355
Female, 36–50, College	3.6219	0.6733	0.094	0.3474	0.093	-0.379	-0.529
Male, 36–50, No College	3.8275	0.0624	0.899	0.0005	7.023	4.041	1.981
Male, 36–50, College	4.1752	0.1018	0.809	0.0018	0.668	-0.149	-0.183
Female, 51–60, No College	3.7038	0.0541	0.792	0.0014	15.385	5.129	9.256
Female, 51–60, College	4.9152	0.0814	0.997	0.0020	***	***	***
Male, 51–60, No College	3.8780	0.0750	0.938	0.0005	40.044	26.647	12.397
Male, 51–60, College	4.4303	0.1281	0.958	0.0011	39.151	33.990	4.161

(C) CF B: Equal Preferences

$\mathbb{E}[\log w]$	Var($\log w$)	Emp	GCI	EV	EV-V	EV-G
4.0762	0.1148	0.693	0.0034	2.663	0.185	1.478
4.4695	0.1758	0.915	0.0031	3.367	1.498	0.869
3.9758	0.0735	0.759	0.0017	1.341	-0.086	0.427
4.4400	0.1454	0.953	0.0019	2.105	0.885	0.220
4.0650	0.0914	0.694	0.0025	1.615	-0.112	0.727
4.2962	0.2066	0.863	0.0017	1.595	0.258	0.337
3.9379	0.0696	0.772	0.0013	0.963	-0.108	0.071
4.1770	0.1032	0.866	0.0013	0.824	-0.122	-0.054
3.9331	0.0913	0.521	0.0084	1.246	-0.289	0.536
3.2847	0.1388	0.417	0.0236	0.607	0.478	-0.871
3.8842	0.0665	0.709	0.0021	0.936	-0.132	0.068
3.9909	0.1157	0.734	0.0026	0.683	-0.146	-0.171

(D) CF C: Equal Match Productivities

k-group	$\mathbb{E}[\log w]$	Var($\log w$)	Emp	GCI	EV	EV-V	EV-G
Female, 26–35, No College	3.6602	0.0942	0.333	0.0433	0.930	-0.088	0.019
Female, 26–35, College	3.5511	0.2040	0.278	0.0737	0.500	-0.220	-0.280
Male, 26–35, No College	3.6964	0.1169	0.665	0.0049	0.659	-0.208	-0.132
Male, 26–35, College	3.6246	0.1306	0.492	0.0133	0.428	-0.350	-0.223
Female, 36–50, No College	3.6927	0.1461	0.422	0.0150	0.654	-0.136	-0.210
Female, 36–50, College	3.6528	0.1934	0.393	0.0221	0.366	-0.301	-0.333
Male, 36–50, No College	3.7047	0.1554	0.899	0.0021	0.528	-0.316	-0.156
Male, 36–50, College	3.6974	0.1668	0.667	0.0022	0.323	-0.491	-0.186
Female, 51–60, No College	3.5801	0.2518	0.333	0.0325	0.631	0.012	-0.382
Female, 51–60, College	3.0738	0.4954	0.395	0.0251	0.482	0.394	-0.912
Male, 51–60, No College	3.6368	0.2100	0.590	0.0036	0.519	-0.224	-0.257
Male, 51–60, College	3.6573	0.2481	0.590	0.0044	0.340	-0.285	-0.374

(E) CF D: Equal Firm Productivity

$\mathbb{E}[\log w]$	Var($\log w$)	Emp	GCI	EV	EV-V	EV-G
3.4832	0.0492	0.317	0.0419	1.019	-0.012	0.031
3.7362	0.0640	0.495	0.0153	1.020	-0.006	0.026
3.6830	0.0540	0.640	0.0033	1.012	-0.022	0.034
3.8697	0.0709	0.718	0.0032	1.003	-0.011	0.015
3.6625	0.0536	0.507	0.0079	1.015	-0.019	0.034
3.9902	0.0853	0.682	0.0038	1.006	-0.011	0.017
3.8436	0.0624	0.744	0.0014	1.004	-0.020	0.024
4.2153	0.1065	0.865	0.0011	0.994	-0.015	0.009
3.6723	0.0615	0.434	0.0151	1.029	-0.037	0.066
4.0204	0.1648	0.476	0.0155	1.040	-0.124	0.165
3.8345	0.0711	0.693	0.0022	1.017	-0.025	0.042
4.2622	0.1518	0.779	0.0019	1.012	-0.021	0.033

(F) Monopsony

TABLE F8. Wage moments, employment, concentration, and welfare by k-group across baseline and counterfactual economies. Columns: $\mathbb{E}[\log w]$ is the employment-weighted mean log wage; Var($\log w$) is the employment-weighted variance of log wages; Emp is the employment rate; GCI is the generalized concentration index; EV is equivalent variation relative to the baseline; EV-V is the component due to changes in the value of matching; EV-G is the component due to changes in concentration. By construction, EV-V + EV-G = EV − 1. Counterfactuals: CF A sets job-amenity utilities to their mean; CF B sets preference parameters (β_k, σ_{kg}) to their means (group 10 omitted as unreliable); CF C sets worker–firm match productivity $(\tilde{\gamma}_{kj}, \rho_k)$ to their means; CF D sets firm productivity and returns-to-scale $(\tilde{\theta}_j, \alpha_j)$ to their medians; Monopsony fixes each firm's labor supply elasticity to $\beta_k\sigma_{kg}$. We exclude one entry for k-group 10 because the estimated wage-preference parameter β for this group is not statistically significant and close to zero, leading to unreliable results.

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